

Math 2143 - Brief Calculus with Applications

Homework #8 - 2008.03.06

Due Date - 2008.03.13

Solutions

Compute the following derivatives.

1.

$$\frac{d}{dt} \left((f(t))^2 \cdot \sqrt{g(t) - h(t)} \right) = 2f(t)f'(t) \cdot \sqrt{g(t) - h(t)} + (f(t))^2 \cdot \frac{g'(t) - h'(t)}{2\sqrt{g(t) - h(t)}}$$

2.

$$\frac{d}{dw} \left((f(w))^2 - (g(w))^4 \right)^2 = 2 \left((f(w))^2 - (g(w))^4 \right) \cdot (2f(w)f'(w) - 4(g(w))^3g'(w))$$

3.

$$\frac{d}{dr} \left(\frac{(h(r))^3 - g(r)}{(g(r))^3 + h(r)} \right) = \frac{(3(h(r))^2h'(r) - g'(r)) \cdot ((g(r))^3 + h(r)) - ((h(r))^3 - g(r)) \cdot (3(g(r))^2g'(r) + h'(r))}{((g(r))^3 + h(r))^2}$$

4.

$$\frac{d}{ds} \left(h \left(\sqrt{f(s)} \cdot g(s) \right) \right) = h' \left(\sqrt{f(s)} \cdot g(s) \right) \cdot \left(\frac{f'(s)}{2\sqrt{f(s)}}g(s) + \sqrt{f(s)}g'(s) \right)$$

5.

$$\frac{d}{dz} (f(g(h(z)) - j(z))) = f'(g(h(z)) - j(z)) \cdot g'(h(z) - j(z)) \cdot (h'(z) - j'(z))$$

For the next five problems, determine which of the above forms the function you are differentiating takes on, write down explicitly what each function is, and compute the derivatives of each 'simple' function. You do not have to put them all together in the final form.

6.

$$\frac{d}{dx} \left(\left(3x^2 - 4x^3 + \frac{1}{x} \right)^2 \sqrt{\frac{4x+1}{4x-1} - \frac{3x+2}{7x^2+2x-1}} \right)$$

This is in the form for problem 1. We have the following:

$$f(x) = \left(3x^2 - 4x^3 + \frac{1}{x} \right), \quad g(x) = \frac{4x+1}{4x-1}, \quad h(x) = \frac{3x+2}{7x^2+2x-1}$$

We take the corresponding derivatives:

$$f'(x) = 6x - 12x^2 - \frac{1}{x^2}, \quad g'(x) = \frac{4 \cdot (4x-1) - (4x+1) \cdot 4}{(4x-1)^2}, \quad h'(x) = \frac{3 \cdot (7x^2+2x-1) - (3x+2) \cdot (14x+2)}{(7x^2+2x-1)^2}$$

7.

$$\frac{d}{dx} \left(\frac{(4x-1)^{\frac{3}{2}} - \left(3x^2 + 2x - \frac{1}{x^2} \right)}{\left(3x^2 + 2x - \frac{1}{x^2} \right)^3 + \sqrt{4x-1}} \right)$$

This is in the form for problem 3. We have the following:

$$h(x) = \sqrt{4x-1}, \quad g(x) = 3x^2 + 2x - \frac{1}{x^2}$$

We take the corresponding derivatives:

$$h'(x) = \frac{4}{2\sqrt{4x-1}}, \quad g'(x) = 6x + 2 + \frac{2}{x^3}$$

8.

$$\frac{d}{dx} \left(\frac{\sqrt{4x-1} \cdot (3x-2) - (\sqrt{4x-1} \cdot (3x-2))^2}{\sqrt{4x-1} \cdot (3x-2) + 1} \right)$$

This is in the form for problem 4. We have the following:

$$f(x) = 4x - 1, \quad g(x) = 3x - 2, \quad h(x) = \frac{x - x^2}{x + 1}$$

We take the corresponding derivatives:

$$f'(x) = 4, \quad g'(x) = 3, \quad h'(x) = \frac{(1 - 2x) \cdot (x + 1) - (x - x^2) \cdot 1}{(x + 1)^2}$$

9.

$$\frac{d}{dx} \left((\sqrt{4x-1} + 1)^2 - \left(\frac{3x+2}{7x-1} \right)^4 \right)^2$$

This is in the form for problem 2. We have the following:

$$f(x) = \sqrt{4x-1} + 1, \quad g(x) = \frac{3x+2}{7x-1}$$

We take the corresponding derivatives:

$$f'(x) = \frac{4}{2\sqrt{4x-1}}, \quad g'(x) = \frac{3 \cdot (7x-1) - (3x+2) \cdot 7}{(7x-1)^2}$$

10.

$$\frac{d}{dx} \left(\frac{1 + \left((3x^2 - \frac{7}{x^2})^2 + (3x^2 - \frac{7}{x^2}) \right)}{1 - \left((3x^2 - \frac{7}{x^2})^2 + (3x^2 - \frac{7}{x^2}) \right)} \right)$$

This is in the form for problem 5. We have the following:

$$f(x) = \frac{1+x}{1-x}, \quad g(x) = x^2 + x, \quad h(x) = 3x^2, \quad j(x) = \frac{7}{x^2}$$

We take the corresponding derivatives:

$$f'(x) = \frac{1 \cdot (1-x) - (1+x) \cdot (-1)}{(1-x)^2}, \quad g'(x) = 2x + 1, \quad h'(x) = 6x, \quad j'(x) = -\frac{14}{x^3}$$