Math 2143 - Brief Calculus with Applications

Extra Limit Problems

Solutions

Compute the following limits.

1.

$$\begin{aligned}\lim_{x\to 0}\left(\frac{1}{3x}-\frac{1}{x(x+3)}\right) &= \lim_{x\to 0}\left(\frac{x+3}{3x(x+3)}-\frac{3}{3x(x+3)}\right)\\ &= \lim_{x\to 0}\frac{x+3-3}{3x(x+3)}\\ &= \lim_{x\to 0}\frac{x}{3x(x+3)}\\ &= \lim_{x\to 0}\frac{1}{3(x+3)}\\ &= \frac{1}{9}\end{aligned}$$

2.

$$\begin{aligned}\lim_{s\to 0}\frac{1-\sqrt{s^2+1}}{s^2} &= \lim_{s\to 0}\frac{1-\sqrt{s^2+1}}{s^2}\cdot\frac{1+\sqrt{s^2+1}}{1+\sqrt{s^2+1}}\\ &= \lim_{s\to 0}\frac{1-(s^2+1)}{s^2\left(1+\sqrt{s^2+1}\right)}\\ &= \lim_{s\to 0}\frac{-s^2}{s^2\left(1+\sqrt{s^2+1}\right)}\\ &= \lim_{s\to 0}\frac{-1}{1+\sqrt{s^2+1}}\\ &= -\frac{1}{2}\end{aligned}$$

3.

$$\begin{aligned}\lim_{x\to 1}\frac{x^6-64}{x-2} &= \lim_{x\to 1}\frac{(x-2)\left(x^5+2x^4+4x^3+8x^2+16x+32\right)}{x-2}\\ &= \lim_{x\to 1}\left(x^5+2x^4+4x^3+8x^2+16x+32\right)\\ &= 6\cdot 2^5 = 192\end{aligned}$$

4.

$$
\begin{aligned}
\lim_{x\to 0}\frac{\sqrt{(1+ax)(1+bx)}-1}{x} &= \lim_{x\to 0}\frac{\sqrt{(1+ax)(1+bx)}-1}{x}\cdot\frac{\sqrt{(1+ax)(1+bx)}+1}{\sqrt{(1+ax)(1+bx)}+1} \\
&= \lim_{x\to 0}\frac{(1+ax)(1+bx)-1}{x\left(\sqrt{(1+ax)(1+bx)}+1\right)} \\
&= \lim_{x\to 0}\frac{\left(1+(a+b)x+abx^2\right)-1}{x\left(\sqrt{(1+ax)(1+bx)}+1\right)} \\
&= \lim_{x\to 0}\frac{(a+b)x+abx^2}{x\left(\sqrt{(1+ax)(1+bx)}+1\right)} \\
&= \lim_{x\to 0}\frac{x((a+b)+abx)}{x\left(\sqrt{(1+ax)(1+bx)}+1\right)} \\
&= \lim_{x\to 0}\frac{(a+b)+abx}{\left(\sqrt{(1+ax)(1+bx)}+1\right)} \\
&= \frac{a+b}{2}
\end{aligned}
$$

5.

$$
\begin{aligned}
\lim_{t\to 2}\frac{t^2-4}{3-\sqrt{t^2+5}} &= \lim_{t\to 2}\frac{t^2-4}{3-\sqrt{t^2+5}}\cdot\frac{3+\sqrt{t^2+5}}{3+\sqrt{t^2+5}} \\
&= \lim_{t\to 2}\frac{\left(t^2-4\right)\left(3+\sqrt{t^2+5}\right)}{9-(t^2+5)} \\
&= \lim_{t\to 2}\frac{\left(t^2-4\right)\left(3+\sqrt{t^2+5}\right)}{-(t^2-4)} \\
&= \lim_{t\to 2}-\left(3+\sqrt{t^2+5}\right) \\
&= -6
\end{aligned}
$$

6.

$$
\begin{aligned}
\lim_{x\to 2}\frac{\sqrt{2}-\sqrt{x}}{x-2} &= \lim_{x\to 2}\frac{\sqrt{2}-\sqrt{x}}{x-2}\cdot\frac{\sqrt{2}+\sqrt{x}}{\sqrt{2}+\sqrt{x}} \\
&= \lim_{x\to 2}\frac{\sqrt{2}-\sqrt{x}}{x-2}\cdot\frac{\sqrt{2}+\sqrt{x}}{\sqrt{2}+\sqrt{x}} \\
&= \lim_{x\to 2}\frac{2-x}{(x-2)\left(\sqrt{2}+\sqrt{x}\right)} \\
&= \lim_{x\to 2}\frac{-1}{\sqrt{2}+\sqrt{x}} \\
&= -\frac{1}{2\sqrt{2}}
\end{aligned}
$$

7.

$$
\begin{aligned}
\lim_{x\to 0^+}\left(\frac{1}{\sqrt{x}}-\frac{1}{\sqrt{x^2+x}}\right) &= \lim_{x\to 0^+}\left(\frac{1}{\sqrt{x}}\cdot\frac{\sqrt{x+1}}{\sqrt{x+1}}-\frac{1}{\sqrt{x+1}\sqrt{x}}\right)\\
&= \lim_{x\to 0^+}\frac{\sqrt{x+1}-1}{\sqrt{x+1}\sqrt{x}}\\
&= \lim_{x\to 0^+}\frac{\sqrt{x+1}-1}{\sqrt{x+1}\sqrt{x}}\cdot\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\\
&= \lim_{x\to 0^+}\frac{x}{\sqrt{x+1}\sqrt{x}\left(\sqrt{x+1}+1\right)}\\
&= \lim_{x\to 0^+}\frac{\sqrt{x}}{\sqrt{x+1}\left(\sqrt{x+1}+1\right)}\\
&= 0
\end{aligned}
$$

8.

$$
\begin{aligned}
\lim_{h\to 0}\frac{\frac{1}{(x+h)^3}-\frac{1}{x^3}}{h} &= \lim_{h\to 0}\frac{\frac{x^3-(x+h)^3}{x^3(x+h)^3}}{h}\\
&= \lim_{h\to 0}\frac{x^3-(x^3+3x^2h+3xh^2+h^3)}{hx^3(x+h)^3}\\
&= \lim_{h\to 0}\frac{-h(3x^2-3xh-h^2)}{hx^3(x+h)^3}\\
&= \lim_{h\to 0}\frac{-(3x^2-3xh-h^2)}{x^3(x+h)^3}\\
&= \frac{-3x^2}{x^3x^3}\\
&= -\frac{3}{x^4}
\end{aligned}
$$

9.

$$
\begin{aligned}
\lim_{p\to 4}\frac{2p^2-32}{p^3-4p^2} &= \lim_{p\to 4}\frac{2(p^2-16)}{p^2(p-4)}\\
&= \lim_{p\to 4}\frac{2(p-4)(p+4)}{p^2(p-4)}\\
&= \lim_{p\to 4}\frac{2(p+4)}{p^2}\\
&= \frac{16}{16}=1
\end{aligned}
$$

10.

$$
\begin{aligned}
\lim_{y\to 0}\left(\frac{1}{y}\left(\frac{1}{4+y}-\frac{1}{4}\right)\right) &= \lim_{y\to 0}\left(\frac{1}{y}\left(\frac{4-(4+y)}{4(4+y)}\right)\right)\\
&= \lim_{y\to 0}\frac{-y}{4y(4+y)}\\
&= \lim_{y\to 0}\frac{-1}{4(4+y)}\\
&= -\frac{1}{16}
\end{aligned}
$$