

Math 2143 - Brief Calculus with Applications

Quiz #5 - 2008.02.26

Solutions

1. Compute the following limit:

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h} \\ \lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h} &= \lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h} \cdot \frac{\sqrt{1+h} + 1}{\sqrt{1+h} + 1} \\ &= \lim_{h \rightarrow 0} \frac{1+h-1}{h(\sqrt{1+h}+1)} \\ &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{1+h}+1)} \\ &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{1+h}+1} \\ &= 1 \cdot \lim_{h \rightarrow 0} \frac{1}{\sqrt{1+h}+1} \\ &= \frac{1}{2}\end{aligned}$$

2. Write down the equation for the slope of the tangent line to the function $f(x) = \sqrt{1+x}$ at $x = 0$.

The slope is given by

$$m = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h}$$

3. Find an equation of the tangent line to $f(x) = \sqrt{1+x}$ and $x = 0$.

We have the slope $m = \frac{1}{2}$ and a point $(0, 1)$. So the equation is

$$y - 1 = \frac{1}{2}(x - 0) \longrightarrow y = \frac{1}{2}x + 1$$