Math 2143 - Brief Calculus with Applications

Quiz #6 - 2008.03.05

Solutions

Consider the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$.

1. Find all the critical points of $f(x)$.

First we take a derivative:

$$f'(x) = 12x^3 - 23x^2 - 24x,$$

which factors as

$$f'(x) = 12x(x^2 - x - 2) = 12x(x - 2)(x + 1).$$

So critical points are located at $x = 0$, $x = -1$ and $x = 2$.

2. Determine the intervals of increase and decrease for $f(x)$.

We break up the real line into the intervals $(-\infty, -1)$, $(-1, 0)$, $(0, 2)$ and $(2, \infty)$.

If we pick a value of x such that $x < -1$, we see that $f'(x) < 0$, so $f(x)$ is decreasing on $(-\infty, -1)$.

Picking an x such that $-1 < x < 0$ gives $f'(x) > 0$ so $f(x)$ is increasing on the interval $(-1, 0)$.

Picking an x such that $0 < x < 2$ (such as $x = 1$) gives $f'(x) < 0$ so $f(x)$ is decreasing on the interval $(0, 2)$.

Finally, for $x > 2$, $f'(x) > 0$ so $f(x)$ is increasing on the interval $(2, \infty)$.