Math 2143 - Brief Calculus with Applications

Quiz #7 - 2008.03.10

Solutions

Consider the function

$$f(x) = \frac{x + \frac{1}{2}}{x^2 + 2}.$$

1. Find all the critical points of $f(x)$.

First we find $f'(x)$:

$$f'(x) = -\frac{x^2 + x - 2}{(x^2 + 2)^2},$$

and then notice that the denominator is never zero. Therefore, setting the numerator to zero gives $(x+2)(x-1) = 0$ or $x = -2$ and $x = 1$.

2. Determine the intervals of increase and decrease for $f(x)$.

We rewrite $f'(x)$ in a nicer fashion first:

$$f'(x) = -\frac{1}{(x^2 + 2)^2} \cdot (x + 2) \cdot (x - 1).$$

The first term is always negative, so we need only determine the sign of the last two terms. Our intervals are $(-\infty, -2)$, $(-2, 1)$ and $(1, \infty)$. For $(-\infty, -2)$, we have that $f'(x) < 0$, for $(-2, 1)$, $f'(x) > 0$ and lastly, $f'(x) < 0$ on $(1, \infty)$.

$f(x)$ is increasing on $(-2, 1)$, and decreasing on $(-\infty, -2)$ and $(1, \infty)$.

3. Classify each critical point as local max, local min or neither.

Since $f'(x)$ goes from positive to negative, and thus $f(x)$ goes from decreasing to increasing, at $x = -2$. We conclude that $f(x)$ has a local min at $x = -2$. In a similar fashion, $f'(x)$ goes from negative to positive, which means that $f(x)$ goes from increasing to decreasing, at $x = 1$. We conclude that $f(x)$ has a local max at $x = 1$