Math 2143 - Brief Calculus with Applications

Quiz #9 - 2008.04.16

Solutions

Answer the following questions assuming that y is a function of x, and satisfies the following implicitly defined equation:

$$x^2 + xy + y^2 = 4$$

1. Show that the point $\left(\frac{2}{\sqrt{3}}, -\frac{4}{\sqrt{3}}\right)$ satisfies the above relation.

It is a straightforward calculation to check that

$$\left(\frac{2}{\sqrt{3}}\right)^2 - \frac{8}{3} + \left(-\frac{4}{\sqrt{3}}\right)^2 = 4.$$

2. Find $\frac{dy}{dx}$.

Differentiating we have

$$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0,$$

and solving for $\frac{dy}{dx}$ gives

$$\frac{dy}{dx} = -\frac{2x + y}{x + 2y}.$$

3. Evaluate $\frac{dy}{dx}$ at the point from problem 1.

$$\frac{dy}{dx}\Big|_{\left(x=\frac{2}{\sqrt{3}}, y=-\frac{4}{\sqrt{3}}\right)} = 0.$$