Math 2283 - Introduction to Logic

Quiz #11 - 2008.11.24

Solutions

Consider the following relation D defined as follows:

$$xDy = x - y, \quad \text{in the case that } x \geq y$$
$$xDy = y - x, \quad \text{in the case that } x \leq y$$

Definitions to remember:

An operation O is commutative if for any x and y: $xOy = yOx$.
An operation O is associative if for any x, y and z: $xO(yOz) = (xOy)Oz$.
An operation O is right-invertible if for any x, y one can find a z such that : $x = yOz$.
An operation O is left-invertible if for any x, y one can find a z such that : $x = zOy$.

1) Is the operation D commutative?

Yes, clearly $xGy = yGx$ since all we do is take the larger of x and y and subtract the smaller one off, it does not matter which order x and y appear in the formula xGy.

2) Is the operation D associative?

No, the operation is not associative. As an example, consider $x = 10$, $y = 5$ and $z = 7$. Then $10D(5D7) = 10D2 = 8$ while $(10D5)D7 = 5D7 = 2$.

3) Is the operation D left- or right-invertible?

Let us first look at right-invertible. The question is, given an arbitrary pair of numbers x and y, can we find a z such that $x = yDz$? The answer is no. Consider the case of $x = -1$ and $y = 0$. Then we have the following expression that we need to solve for z in: $-1 = 0Dz$. The only potential values that would work are $z = 1$ and $z = -1$. But $0D(-1) = 1$ and $0D1 = 1$. Similarly, D is also not right invertible by the same reason (and that D is commutative).