Math 2283 - Introduction to Logic

Quiz #7 - 2008.10.09

Solutions

Consider the set $\mathbf{D} = \{0, 1, 2\}$ and define the class $\mathbb{K}$ to be the class of all possible subsets of $\mathbf{D}$, explicitly given as

$$\mathbb{K} = \{\{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}, \{\}\}.$$

We now define the relation $\subseteq$ for two elements x, y of the class $\mathbb{K}$ by:

$$x \subseteq y \leftrightarrow (x \subset y) \vee (x = y).$$

Note: This relation is very similar to the relation of inclusion, however we now have the added property of possible equality.

Determine which of the properties: reflexive, irreflexive, symmetric, asymmetric, transitive and connected, the relation $\subseteq$ has with respect to the class $\mathbb{K}$.

First we check reflexive: $\underset{x}{\mathbf{A}}\ x \subseteq x$. This is true since $\underset{x}{\mathbf{A}}\ x = x$.

Next up is irreflexive: $\underset{x}{\mathbf{A}}\ x \nsubseteq x$. This is false since $\underset{x}{\mathbf{A}}\ x = x$.

Symmetric: $\underset{x,y}{\mathbf{A}}\ x \subseteq y \rightarrow y \subseteq x$. This is false, consider $x = \{1\}$ and $y = \{0, 1\}$.

Asymmetric: $\underset{x,y}{\mathbf{A}}\ x \subseteq y \rightarrow y \nsubseteq x$. This is false, consider $y = x$.

Transitive: $\underset{x,y,z}{\mathbf{A}}\ (x \subseteq y \wedge\ y \subseteq z) \rightarrow\ x \subseteq z$. This is true since $x \subseteq y$ implies everything that is in x is also in y, and $y \subseteq z$ implies that everything in y is also in z. Therefore, all of x must also be in z giving us $x \subseteq z$.

Connected: $\underset{x,y}{\mathbf{A}}\ (x \neq y) \wedge (x \subseteq y \vee y \subseteq x)$. This is false, consider $x = \{0, 2\}$ and $y = \{0, 1\}$.