

Math 2283 - Introduction to Logic

Quiz #8 - 2008.10.29

Name: _____

Consider the following axiomatic system where we assume the symbols \rightarrow , \leftrightarrow (defined as usual) as primitive terms, and these five sentences as axioms:

Ax I: $(p \leftrightarrow q) \rightarrow (q \rightarrow p)$

Ax II: $(p \leftrightarrow q) \rightarrow (p \rightarrow q)$

Ax III: $(p \rightarrow q) \rightarrow [(q \rightarrow r) \rightarrow (p \rightarrow r)]$

Ax IV: $(p \rightarrow q) \rightarrow [(q \rightarrow p) \rightarrow (p \leftrightarrow q)]$

Ax V: $[p \rightarrow (q \rightarrow r)] \rightarrow [q \rightarrow (p \rightarrow r)]$

Furthermore, we agree to apply in the proof of the following theorem only the rule of substitution and the law of detachment (L.O.D.).

Prove the following theorem:

Theorem: $(q \rightarrow p) \rightarrow [(q \leftrightarrow p) \rightarrow (p \leftrightarrow q)]$

Hint 1: Ax III above is the same one that you have been using for the shortcut method described in class, so you can use the shortcut from class if need be.

Hint 2: You will need to swap p and q in either Ax I or Ax II.

Hint 3: The result of Hint 2 and Ax IV should be compared.

Hint 4: The result of Hint 3 should be compared to the theorem that you want to prove, then use an axiom that you have not yet utilized. You will need to do a serious substitution in this axiom.

Hint 5: The L.O.D. will finish the proof of the theorem.