

# Math 2283 - Introduction to Logic

Quiz #9 - 2008.10.31

Name: \_\_\_\_\_

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Consider the following axiomatic system where we assume the symbols  $\rightarrow$ ,  $\leftrightarrow$  (defined as usual) as primitive terms, and these five sentences as axioms:

Ax I:  $p \rightarrow (q \rightarrow p)$

Ax II:  $(p \leftrightarrow q) \rightarrow (p \rightarrow q)$

Ax III:  $(p \rightarrow q) \rightarrow [(q \rightarrow r) \rightarrow (p \rightarrow r)]$

Ax IV:  $[p \rightarrow (q \rightarrow r)] \rightarrow [q \rightarrow (p \rightarrow r)]$

Furthermore, we agree to apply in the proof of the following theorem only the rule of substitution and the law of detachment (L.O.D.).

Prove the following theorem:

Theorem:  $(p \leftrightarrow q) \rightarrow [p \rightarrow (p \rightarrow q)]$

Hint 1: Ax III above is the same one that you have been using for the shortcut method described in class, so you can use the shortcut from class if need be.

Hint 2: Applying Ax III (shortcut) to Ax II and Ax III is a good first step.

Hint 3: Ax IV should be used next with a suitable substitution.

Hint 4: Now find a suitable substitution for  $r$  in the result of hint 3.

Hint 5: If done correctly, the antecedent of the result of hint 4 should look like Ax I after swapping variables in Ax I.