Math 2315 - Calculus II

Homework #10 - 2007.11.02

Due Date - 2007.11.09

Name: ____________________________

Part 1: Problems from sections 11.1 - 11.2

Part 2: The *fun* problems.

1. If $a_n = \sqrt{n+3} - \sqrt{n}$, compute $\lim_{n\to\infty} a_n$.

2. If $b_n = n^2\left(\sqrt[3]{n^3+1} - n\right)$, compute $\lim_{n\to\infty} b_n$.

3. Let $c_n = \frac{\sqrt[n]{n!}}{n}$

   a) Show that
$$\ln(c_n) = \frac{\ln(n!) - n\ln(n)}{n}.$$

   b) Show that
$$\frac{\ln(n!) - n\ln(n)}{n} = \frac{1}{n}\sum_{k=1}^{n} \ln\left(\frac{k}{n}\right).$$

   c) Show that $\ln(c_n)$ converges to $\int_0^1 \ln(x)dx$.

   d) Prove that $\lim_{n\to\infty} c_n = \frac{1}{e}$.

4. Prove that if a is a positive integer, then
$$\sum_{n=1}^{\infty} \frac{1}{n(n+a)} = \frac{1}{a}\left(1 + \frac{1}{2} + \cdots + \frac{1}{a}\right).$$

5. A ball dropped from a height of 100 ft begins to bounce. Each time it strikes the ground, it returns to two-thirds of its previous height. What is the total distance traveled by the ball if it bounces infinitely many times?

6. If $\sum_{k=1}^{\infty} a_k$ is convergent, is it necessarily true that adding or removing a finite number of terms to the sum must yield a series which is also convergent? If not, give an example to prove your point.

7. Prove that

$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+a} = \sum_{n=a+1}^{\infty} \frac{1}{n-a} - \frac{1}{n}.$$