# Math 2315 - Calculus II

**Homework #1 - 2007.08.23**

**Due Date - 2007.08.30**

**Solutions**

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Part 1: Problems from sections 7.1 and 7.2.
Section 7.1:

17.

$$\int y\sinh(y)dy$$

Setting $f = y$ and $g' = \sinh(y)$, then $f' = 1$ and $g = \cosh(y)$. So we have

$$\int y\sinh(y)dy = y\cosh(y) - \sinh(y) + C.$$

31.

$$\int_1^2 x^4 \left(\ln(x)\right)^2 dx$$

Setting $f' = x^4$ and $g = \left(\ln(x)\right)^2$, this gives $f = \frac{1}{5}x^5$ and $g' = \frac{2}{x}\ln(x)$. So

$$\int_1^2 x^4 \left(\ln(x)\right)^2 dx = \frac{1}{5}x^5 \left(\ln(x)\right)^2 \Big|_1^2 - \int_1^2 \frac{2}{5}x^4 \ln(x)dx.$$

So now we focus on the integral on the right hand side. Setting $f' = x^4$ and $g = \ln(x)$ we get

$$\begin{aligned}
\int_1^2 x^4 \left(\ln(x)\right)^2 dx &= \frac{1}{5}x^5 \left(\ln(x)\right)^2 \Big|_1^2 - \frac{2}{5}\int_1^2 x^4 \ln(x)dx \\
&= \frac{1}{5}x^5 \left(\ln(x)\right)^2 \Big|_1^2 - \frac{2}{5}\left[\frac{1}{5}x^5 \ln(x)\Big|_1^2 - \int_1^2 x^4 dx\right] \\
&= \frac{1}{5}x^5 \left(\ln(x)\right)^2 \Big|_1^2 - \frac{2}{5}\left[\frac{1}{5}x^5 \ln(x)\Big|_1^2 - \frac{1}{5}x^5\Big|_1^2\right] \\
&= \frac{32}{5}\ln(2)^2 - \frac{64}{25}\ln(2) + \frac{62}{125}.
\end{aligned}$$

Section 7.2:

31.

$$\int \tan^5(x)dx$$

First we rewrite as

$$\begin{aligned}\int \tan^5(x)dx &= \int \left(1 - \sec^2(x)\right)^2 \tan(x)dx \\ &= \int \left(1 - 2\sec^2(x) + \sec^4(x)\right)\tan(x)dx \\ &= \int \tan(x)dx - 2\int \sec^2(x)\tan(x)dx + \int \sec^4(x)\tan(x)dx\end{aligned}$$

The last two integrals can be solved by substitution. In particular, set $u = \sec(x)$ and then $du = \sec(x)\tan(x)dx$. After substitution, we get

$$\int \tan^5(x)dx = \ln\left(|\sec(x)|\right) - \sec^2(x) + \frac{1}{4}\sec^4(x) + C.$$

46.

$$\int \frac{dx}{\cos(x) - 1}dx$$

First we do some trigonometric magic:

$$1 - \cos(x) = 2\sin^2\left(\frac{x}{2}\right),$$

so we get

$$\int \frac{dx}{\cos(x) - 1}dx = -\frac{1}{2}\int \csc^2\left(\frac{x}{2}\right)dx.$$

This of course has an easy antiderivative, so we get

$$\int \frac{dx}{\cos(x) - 1}dx = \cot\left(\frac{x}{2}\right) + c.$$

Part 2: The *fun* problems.

1. Compute

$$\int x^k \ln(x)dx, \quad k \neq -1$$

Integrating by parts, we set $u = x^k$, $v' = \ln(x)$, giving $u' = kx^{k-1}$ and $v = x\ln(x) - x$. So

$$\begin{aligned}\int x^k \ln(x)dx &= x^{k+1}\ln(x) - x^{k+1} - \int kx^k \ln(x) - kx^k dx \\ &= x^{k+1}\ln(x) - x^{k+1} - \int kx^k \ln(x)dx + \int kx^k dx.\end{aligned}$$

Moving the first integral on the right to the left sides gives

$$(1+k)\int x^k \ln(x)dx = x^{k+1}\ln(x) - x^{k+1} + \frac{k}{k+1}x^{k+1},$$

and solving for the integral gives

$$\int x^k \ln(x)dx = \frac{1}{k+1}x^{k+1}\ln(x) - \frac{1}{(k+1)^2}x^{k+1}$$

2. Compute

$$\int_{-1}^{1} t^5 \cos^6(2t)dt$$

This function is odd and is being integrating about an even interval, so we can conclude that

$$\int_{-1}^{1} t^5 \cos^6(2t)dt = 0.$$

3. Compute

$$\int xe^x \cos(x)dx$$

There are several ways to do this, we are going to do the following: Setting $u = x$ and $v' = e^x \cos(x)$, we get $u' = 1$ and $v = \frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)$, which is what we calculated in class. So we have

$$\begin{aligned}\int xe^x \cos(x)dx &= \frac{1}{2}xe^x \cos(x) + \frac{1}{2}xe^x \sin(x) - \frac{1}{2}\int e^x \cos(x) + e^x \sin(x)dx \\ &= \frac{1}{2}xe^x \cos(x) + \frac{1}{2}xe^x \sin(x) - \frac{1}{2}\left[\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)\right] \\ &\quad - \frac{1}{2}\int e^x \sin(x)dx.\end{aligned}$$

To compute the integral on the right hand side, we can simply apply the same method that was used on the $e^x \cos(x)$ integral. This gives

$$\int e^x \sin(x)dx = -\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x).$$

So we finish with

$$\begin{aligned}\int xe^x \cos(x)dx =& \frac{1}{2}xe^x \cos(x) + \frac{1}{2}xe^x \sin(x) - \frac{1}{2}\left[\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)\right] \\ &- \frac{1}{2}\left[-\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)\right] \\ =& \frac{1}{2}xe^x \cos(x) - \left(-\frac{1}{2}x + \frac{1}{2}\right) e^x \sin(x).\end{aligned}$$

4. Compute

$$\int x\cos^3(x)\sin^2(x)dx$$

We start with $u = x$ and $v' = \cos^3(x)\sin^2(x)$. Clearly, $u' = 1$, but we need to compute $v$:

$$\int \cos^3(x)\sin^2(x)dx = \int \left(1-\sin^2(x)\right)\sin^2(x)\cos(x)dx.$$

Setting $U = \sin(x)$ gives $dU = \cos(x)dx$ and thus

$$\begin{aligned}\int \left(1-\sin^2(x)\right)\sin^2(x)\cos(x)dx &= \int \left(1-U^2\right)U^2dU \\ &= \int U^2 - U^4 dU \\ &= \frac{1}{3}U^3 - \frac{1}{5}U^5 \\ &= \frac{1}{3}\sin^3(x) - \frac{1}{5}\sin(x)^5\end{aligned}$$

So now we have $v = \frac{1}{3}\sin^3(x) - \frac{1}{5}\sin(x)^5$, so integration by parts yields

$$\int x\cos^3(x)\sin^2(x)dx = \frac{1}{3}x\sin^3(x) - \frac{1}{5}x\sin(x)^5 - \int \frac{1}{3}\sin^3(x) - \frac{1}{5}\sin(x)^5dx.$$

So we have to compute the two following integrals:

$$\int \frac{1}{3}\sin^3(x)dx, \quad \int -\frac{1}{5}\sin(x)^5dx.$$

We work on the first:

$$\int \frac{1}{3}\sin^3(x)dx = \frac{1}{3}\int \left(1-\cos^2(x)\right)\sin(x)dx,$$

by setting $U = \cos(x)$, we get

$$\begin{aligned}\frac{1}{3}\int \left(1-\cos^2(x)\right)\sin(x)dx &= -\frac{1}{3}\int 1 - u^2du \\ &= -\frac{1}{3}\left(u - \frac{1}{3}u^3\right) \\ &= -\frac{1}{3}\cos(x) + \frac{1}{9}\cos^3(x)\end{aligned}$$

and

$$\begin{aligned}
\int -\frac{1}{5}\sin^5(x)dx &= -\frac{1}{5}\int \left(1-\cos^2(x)\right)^2 \sin(x)dx \\
&= \frac{1}{5}\int (1-u^2)^2 du \\
&= \frac{1}{5}\int \left(1-2u^2+u^4\right) du \\
&= \frac{1}{5}\left(u-\frac{2}{3}u^3+\frac{1}{5}u^5\right) \\
&= \frac{1}{5}\left[\cos(x)-\frac{2}{3}\cos^3(x)+\frac{1}{5}\cos^5(x)\right].
\end{aligned}$$

Putting this all together gives

$$\begin{aligned}
\int x\cos^3(x)\sin^2(x)dx =& \frac{1}{3}x\sin^3(x)-\frac{1}{5}x\sin(x)^5+\frac{1}{3}\cos(x)-\frac{1}{9}\cos^3(x) \\
&-\frac{1}{5}\left(\cos(x)-\frac{2}{3}\cos^3(x)+\frac{1}{5}\cos^5(x)\right) \\
=& \frac{1}{3}x\sin^3(x)-\frac{1}{5}x\sin(x)^5 \\
&+\frac{2}{15}\cos(x)+\frac{1}{45}\cos^3(x)-\frac{1}{25}\cos^5(x)
\end{aligned}$$