Math 2315 - Calculus II

Homework #3 - 2007.09.02

Due Date - 2007.09.12

Name: ____________________________

Part 1: Problems from sections 7.5.

Part 2: The *fun* problems.

1. Solve the following two hyperbolic integrals using a similar method to trigonometric integration.

a)

$$\int \sinh^4(x)\cosh^5(x)dx$$

b)

$$\int \cosh^2(x)dx$$

2. Suppose that $Q(x) = (x-a)(x-b)$, where $a \neq b$ and let $\frac{P(x)}{Q(x)}$ be a proper rational function so that

$$\frac{P(x)}{Q(x)} = \frac{A}{x-a} + \frac{B}{x-b}.$$

Show that

$$A = \frac{P(a)}{Q'(a)} \text{ and } B = \frac{P(b)}{Q'(b)}.$$

3. Let us now try to generalize problem 2 a little bit. Suppose that

$$Q(x) = (x-a_1)(x-a_2)\cdots(x-a_n) = \prod_{k=1}^{n}(x-a_k),$$

where $a_i \neq a_j$ for $1 \leq i, j, \leq n$. Let $\frac{P(x)}{Q(x)}$ be a proper rational function so that

$$\frac{P(x)}{Q(x)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + \cdots + \frac{A_n}{x-a_n} = \sum_{k=1}^{n}\frac{A_k}{x-a_k}.$$

The goal of this problem is to show that $A_j = \frac{P(a_j)}{Q'(a_j)}$. To make this a little easier, we will break it up a little bit. For the rest of this problem, we will thus assume that $Q(x) = \prod_{k=1}^{n}(x - a_k)$, as already stated.

a) Show that

$$Q'(x) = Q(x) \cdot \sum_{k=1}^{n} \frac{1}{x - a_k}.$$

b) Using part a) show that

$$Q'(a_j) = \prod_{k=1, k \neq j}^{n} (a_j - a_k)$$

c) From the definition of $Q(x)$ and parts a) and b), show that

$$Q^2(a_j) \cdot \sum_{k=1}^{n} \frac{A_k}{(a_j - a_k)^2} = \left(Q'(a_j)\right)^2 A_j.$$

d) Using parts a)-c) and your method of proving problem 2), show that

$$A_j = \frac{P(a_j)}{Q'(a_j)}.$$