Math 2315 - Calculus II

Homework #6 - 2007.09.25

Due Date - 2007.10.01

Name: _____

Part 1: Problems from sections 6.5. and 7.8

Part 2: The *fun* problems.

1. If for any $x \geq 0$ the average value of $R(x)$ on the interval $[0, x]$ is equal to x, find $R(x)$.

2. The average value of $g(t)$ on $[2, 5]$ is equal to 9, find $\int_2^5 g(t)dt$.

3. Consider the following improper integral:

$$I = \int_4^\infty \frac{dx}{(x-2)(x-3)}.$$

a) Show that if $R > 4$

$$\int_4^R \frac{dx}{(x-2)(x-3)} = \ln\left(\left|\frac{R-3}{R-2}\right|\right) - \ln\left(\frac{1}{2}\right).$$

b) Show that $I = \ln(2)$.

4. Assuming that $a > 0$,

a) Show that $\lim_{x\to\infty} \frac{x^a}{\ln(x)} = \infty$

b) Show that $x^a > 2\ln(x)$ for x sufficiently large.

c) Show that $e^{-x^a} < x^{-2}$ for x sufficiently large.

d) Show that $\int_1^\infty e^{-x^a} dx$ converges.

5. Let S be the solid obtained by rotating the region below the graph of $y = \frac{1}{x}$ about the x-axis for $1 \leq x \leq \infty$.

a) Use the washer method to compute the volume of S. Note that the volume is finite even though S is an infinite region.

b) It can be shown that the surface area of S is

$$SA = 2\pi \int_1^\infty \frac{1}{x}\sqrt{1 + \frac{1}{x^4}} dx.$$

Show that SA is infinite.