Math 2315 - Calculus II

Homework #7 - 2007.10.01

Due Date - 2007.10.08

Name: _____

Part 1: Problems from sections 8.1 and 8.2

Part 2: The *fun* problems.

1. a) Give a geometric argument which shows that the arc length of the curve $y = x^2$ on the interval $[0, a]$ is the same as the arc length of the curve $y = \sqrt{x}$ on the same interval.

b) The arc length of the curve $y = x^2$ on the interval $[0, a]$ is equal to the arc length of the curve $y = \sqrt{x}$ on what interval? You might want to use your answer to part a) to help with part b).

2. Consider the parabola $y = \alpha (x - \beta)^2 - \gamma$. Construct a method to simplify the arc length integral on the interval $[a, b]$ by moving the parabola to the origin. Be sure to show that your resulting integral is indeed equal to the original.

3. Find the value of a such that the arc length of the *catenary* $y = \cosh(x)$ for $-a \leq x \leq a$ equals 10.

4. Find the arc length of

$$y = \left(\frac{x}{2}\right)^4 + \frac{1}{2x^2}$$

over the interval $[1, 4]$.

5. Show that the arc length of $y = \ln(f(x))$ for $a \leq x \leq b$ is given by

$$\int_a^b \frac{\sqrt{(f(x))^2 + (f'(x))^2}}{f(x)} dx.$$

6. Use problem 5 to compute the arc length of $y = \ln(\sin(x))$ for $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$.