

Math 2315 - Calculus II

Homework #8 - 2007.10.10

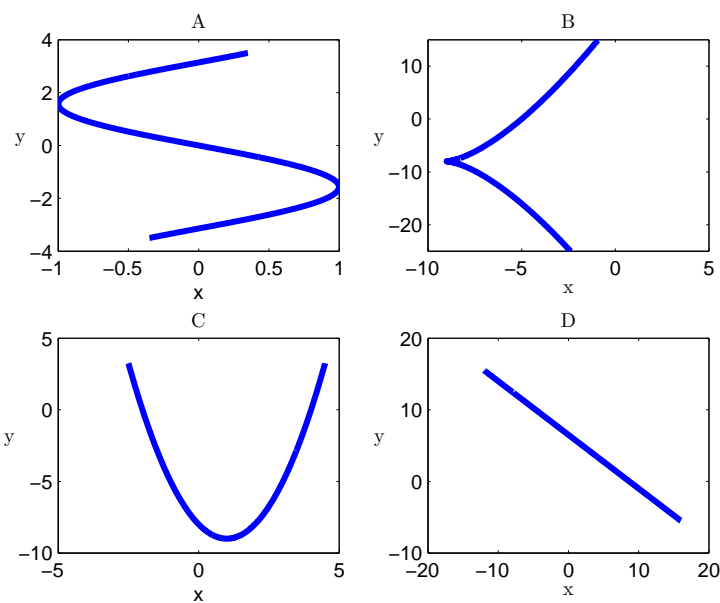
Due Date - 2007.10.17

Name: _____

Part 1: Problems from sections 10.1 and 10.2

Part 2: The *fun* problems.

1. Match the parameterizations (a)-(d) below with their plots in the following figure and draw an arrow indicating the direction of motion.



a) $x(t) = \sin(t), y(t) = -t$

b) $x(t) = t^2 - 9, y(t) = -t^3 - 8$

c) $x(t) = 1 - t, y(t) = t^2 - 9$

d) $x(t) = 4t + 2, y(t) = 5 - 3t$

2. Find a parametrization $c(t) = (x(t), y(t))$ of the line $y = 3x + 4$ such that $c(0) = (2, 10)$.

3. Find $\frac{dy}{dx}$ at $t = -4$ if $(x(t), y(t)) = (t^3, t^2 - 1)$.

4. If $c(t) = (x(t), y(t)) = (\cos(t), \cos(t) + \sin^2(t))$, find an expression for $y = f(x)$ and compute $\frac{dy}{dx}$ both by differentiating $f(x)$ and by differentiating $c(t)$ appropriately.

5. An object moves along the path $x(t) = \frac{1}{4}t^3 + 2t$, $y(t) = 20t - t^2$ where time is measured in seconds and position in feet.

a) What is the maximum height attained by the object?

b) At what time does the object hit the ground?

c) How far is the object from the origin when it hits the ground?

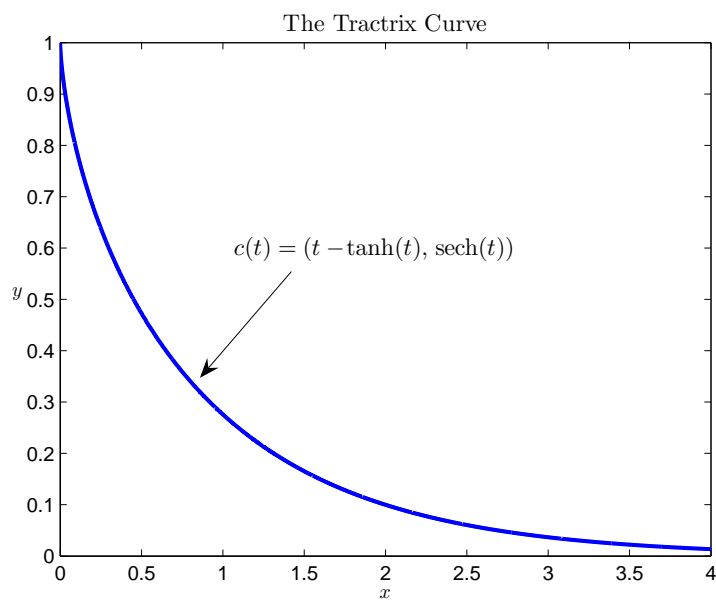
6. A bullet fired from a gun follows the trajectory

$$x(t) = at, \quad y(t) = bt - 16t^2 \quad (a, b > 0).$$

Show that the bullet leaves the gun at an angle of $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ and lands at a distance $\frac{ab}{16}$ from the origin.

7. What is the interpretation of $\sqrt{(x'(t))^2 + (y'(t))^2}$ for a particle following the trajectory $(x(t), y(t))$?

8. The curve $c(t) = (t - \tanh(t), \operatorname{sech}(t))$ is called a *tractrix* (see the figure below). Calculate the surface area of the infinite surface generated by revolving the tractrix about the x -axis for $0 \leq t < \infty$.



9. Verify that the tractrix curve defined in problem 8 has the following property: For all t , the segment from $c(t)$ to $(t, 0)$ is tangent to the curve and has length 1. (See the figure below.)

