# Math 2315 - Calculus II

## Quiz #14 - 2007.12.04

## Solutions

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1. Find the Maclaurin series for $f(x) = \cos(2x)$.

We start with

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!},$$

and replace $x$ with $2x$ to get

$$\cos(2x) = \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} (-4)^n \frac{x^{2n}}{(2n)!}.$$

2. Find the Taylor series for $f(x) = \sin(x)$ centered at $x = \frac{\pi}{4}$. You do not have to write the formula in summation form.

First we notice that $f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$. Next, we compute $f'(x) = \cos(x)$ and $f'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$. Similarly, we have

$$\begin{aligned}
f\left(\frac{\pi}{4}\right) &= \frac{1}{\sqrt{2}} \\
f'\left(\frac{\pi}{4}\right) &= \frac{1}{\sqrt{2}} \\
f''\left(\frac{\pi}{4}\right) &= -\frac{1}{\sqrt{2}} \\
f'''\left(\frac{\pi}{4}\right) &= -\frac{1}{\sqrt{2}} \\
f^{(iv)}\left(\frac{\pi}{4}\right) &= \frac{1}{\sqrt{2}} \\
f^{(v)}\left(\frac{\pi}{4}\right) &= \frac{1}{\sqrt{2}} \\
f^{(vi)}\left(\frac{\pi}{4}\right) &= -\frac{1}{\sqrt{2}} \\
f^{(vii)}\left(\frac{\pi}{4}\right) &= -\frac{1}{\sqrt{2}} \\
&\vdots
\end{aligned}$$

So putting this together gives

$$f(x) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right) - \frac{1}{2!}\frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right)^2 - \frac{1}{3!}\frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right)^3 + \frac{1}{4!}\frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right)^4 + \frac{1}{5!}\frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right)^5 + \dots$$