

Math 2315 - Calculus II

Quiz #1 - 2007.08.29

Solutions

Compute the following integrals:

1.

$$\int x^7 \ln(x) dx$$

We perform integration by parts. Setting $f' = x^7$ and $g = \ln(x)$, we get $f = \frac{1}{8}x^8$ and $g' = \frac{1}{x}$. So we get:

$$\begin{aligned} \int x^7 \ln(x) dx &= \frac{1}{8}x^8 \ln(x) - \int \frac{1}{8}x^8 \frac{1}{x} dx \\ &= \frac{1}{8}x^8 \ln(x) - \int \frac{1}{8}x^7 dx \\ &= \frac{1}{8}x^8 \ln(x) - \frac{1}{64}x^8 + C. \end{aligned}$$

2.

$$\int \sin^{15}(t) \cos^3(t) dt$$

This is a trigonometric integral involving sines and cosines. Notice that the power of the sine term is very dominant. So we are going to let $u = \sin(t)$ and $du = \cos(t)dt$. So first we do some rewriting:

$$\begin{aligned} \int \sin^{15}(t) \cos^3(t) dt &= \int \sin^{15}(t) \cos^2(t) \cos(t) dt \\ &= \int \sin^{15}(t)(1 - \sin^2(t)) \cos(t) dt. \end{aligned}$$

After the substitution, we get

$$\begin{aligned} \int \sin^{15}(t)(1 - \sin^2(t)) \cos(t) dt &= \int u^{15}(1 - u^2) du \\ &= \int u^{15} - u^{17} du \\ &= \frac{1}{16}u^{16} - \frac{1}{18}u^{18} + C \\ &= \frac{1}{16} \sin^{16}(t) - \frac{1}{18} \sin^{18}(t) + C \end{aligned}$$