

# Math 2315 - Calculus II

Quiz #2 - 2007.09.05

Solutions

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Compute the following integral:

$$\int \frac{dx}{\sqrt{x^2 - 9}}$$

We will use the identity  $\sec^2(\theta) - 1 = \tan^2(\theta)$ . So by setting  $x = 3\sec(\theta)$ ,  $dx = 3\sec(\theta)\tan(\theta)d\theta$ , and our integral now becomes:

$$\begin{aligned}\int \frac{dx}{\sqrt{x^2 - 9}} &= \int \frac{3\sec(\theta)\tan(\theta)}{\sqrt{9\sec^2(\theta) - 9}}d\theta \\ &= \int \frac{3\sec(\theta)\tan(\theta)}{3\tan(\theta)}d\theta \\ &= \int \sec(\theta)d\theta \\ &= \ln(\sec(\theta) + \tan(\theta)) + P\end{aligned}$$

Now we need to convert the integral back to the variable  $x$ . Since  $x = 3\sec(\theta)$ , we have that  $\cos(\theta) = \frac{3}{x}$ . This gives (draw a triangle if you have to)  $\tan(\theta) = \frac{\sqrt{x^2 - 9}}{3}$ . So

$$\int \frac{dx}{\sqrt{x^2 - 9}} = \ln\left(\frac{x}{3} + \frac{\sqrt{x^2 - 9}}{3}\right) + P.$$