

Math 2315 - Calculus II

Quiz #4 - 2007.09.14

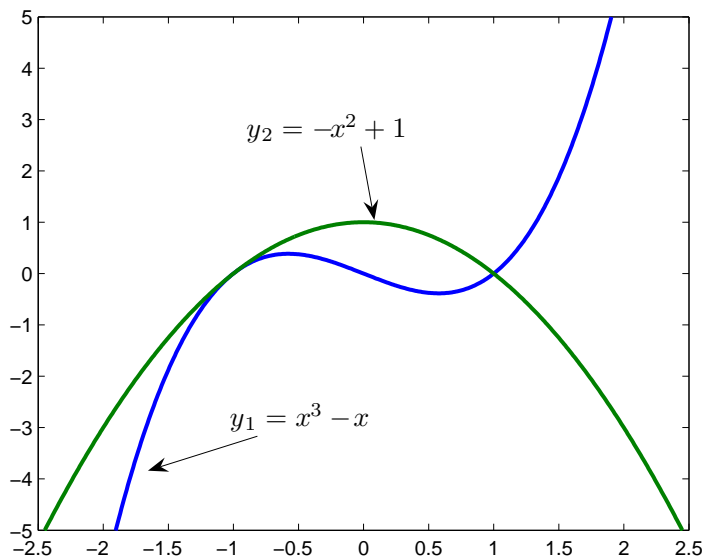
Solutions

Find the area between the two curves $y_1 = x^3 - x$ and $y_2 = -x^2 + 1$.

First we need to determine where the two functions intersect. So we factor:

$$y_1 = x(x-1)(x+1), \quad y_2 = -(x+1)(x-1).$$

Setting these two equal to each other (or noting that they both have roots at $x = 1$ and $x = -1$), we find that the two points of intersection are at $x = -1$ and $x = 1$, where the point $x = -1$ is an intersection point of multiplicity 2. We also know the shape of these graphs, so we get the following picture.



We now set up and compute the definite integral:

$$Area = \int_{-1}^1 (-x^2 + 1 - x^3 + x) dx = -\frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2 + x \Big|_{-1}^1 = \frac{4}{3}.$$