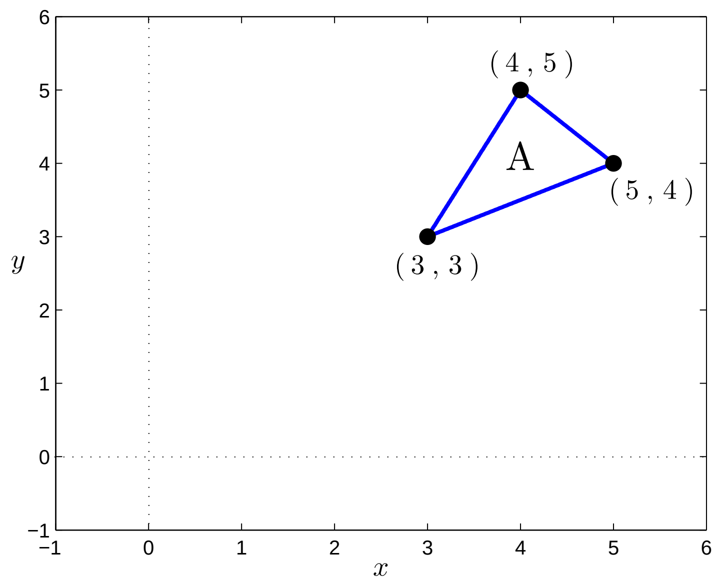
Math 2315 - Calculus II

Quiz #5 - 2007.09.19

Solutions



1. Set up, but do not evaluate, the integrals required to compute, by both the method of cylindrical shells and the washer method, the volume of the solid obtained by rotating the region A about the $x-$axis.

The equation of the line from $(3,3)$ to $(4,5)$ is given by $y_1 = 2x-3$. The equation of the line from $(4,5)$ to $(5,4)$ is given by $y_2 = -x+9$, and finally the equation of the line between the points $(3,3)$ and $(5,4)$ is given by $y_3 = \frac{1}{2}x + \frac{3}{2}$. The washer method:

$$V = \int_3^4 \pi \left[(2x-3)^2 - \left(\frac{1}{2}x + \frac{3}{2}\right)^2\right] dx + \int_4^5 \pi \left[(-x+9)^2 - \left(\frac{1}{2}x + \frac{3}{2}\right)^2\right] dx$$

To use the method of cylindrical shells, we need to solve each line equation for x in terms of y. This gives

$$x_1 = \frac{1}{2}y + \frac{3}{2}, \quad x_2 = 9 - y, \quad x_3 = 2y - 3.$$

The method of cylindrical shells:

$$V = \int_3^4 2\pi\ y\ \left[(2y-3) - \left(\frac{1}{2}y + \frac{3}{2}\right)\right] dy + \int_4^5 2\pi\ y\ \left[(9-y) - \left(\frac{1}{2}y + \frac{3}{2}\right)\right] dy$$

2. Set up, but do not evaluate, the integrals required to compute, by both the method of cylindrical shells and the washer method, the volume of the solid obtained by rotating the region A about the $y-$axis.

Here we have already done all the grunt work in problem 1.The washer method:

$$V = \int_3^4 \pi \left[(2y-3)^2 - \left(\frac{1}{2}y + \frac{3}{2}\right)^2\right] dy + \int_4^5 \pi \left[(9-y)^2 - \left(\frac{1}{2}y + \frac{3}{2}\right)^2\right] dy$$

The method of cylindrical shells:

$$V = \int_3^4 2\pi\ x\ \left[(2x-3) - \left(\frac{1}{2}x + \frac{3}{2}\right)\right] dx + \int_4^5 2\pi\ x\ \left[(9-y) - \left(\frac{1}{2}x + \frac{3}{2}\right)\right] dx$$