Math 2315 - Calculus II

Quiz #6 - 2007.09.21

Solutions

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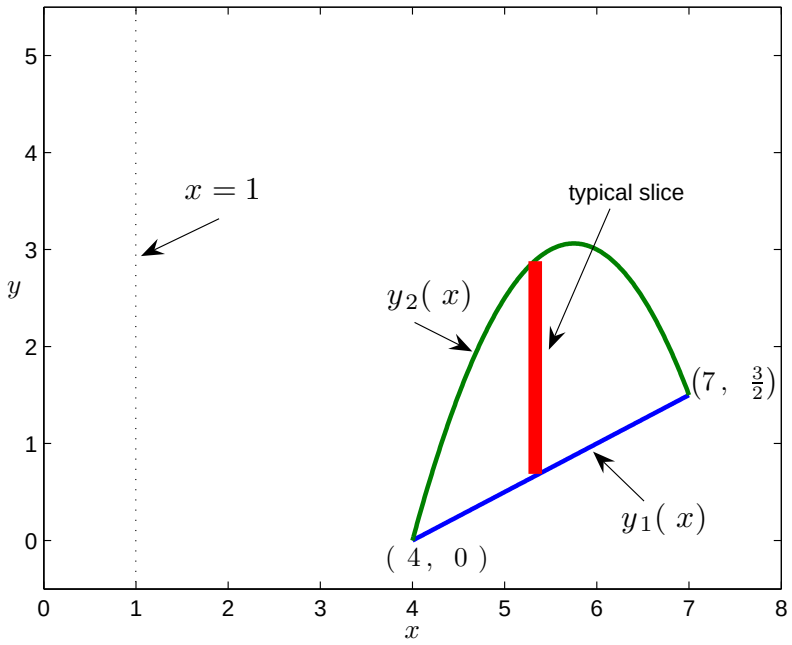
1. Find the volume of the solid obtained by rotating the region bounded by the curves $y_1 = \frac{1}{2}x - 2$ and $y_2 = -(x-4)\left(x - \frac{15}{2}\right)$ about the line $x = 1$.



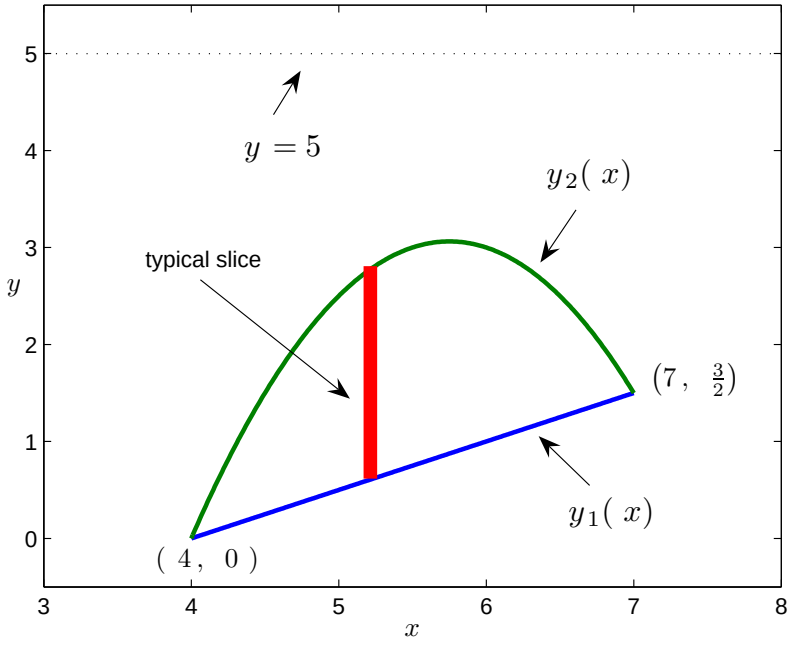
First we draw the functions, the points of intersection and the line about which we are rotating. From inspecting the graph, we know that we need to use the method of cylindrical shells(why?). So we need to determine the height and radius. They are given by:

$$h = y_2 - y_1 = -(x-4)\left(x - \frac{15}{2}\right) - \frac{1}{2}x + 2, \quad r = x - 1.$$

We put this all together to get

$$V = \int_4^7 2\pi\ r\ h\ dx = \int_4^7 2\pi\ (x-1)\ \left(-(x-4)\left(x - \frac{15}{2}\right) - \frac{1}{2}x + 2\right)\ dx = \frac{81}{2}\pi.$$

2. Find the volume of the solid obtained by rotating the region bounded by the curves $y_1 = \frac{1}{2}x - 2$ and $y_2 = -(x-4)\left(x - \frac{15}{2}\right)$ about the line $y = 5$.



We have a very similar picture here, except we are now revolving around the $y = 5$ line. Our integral will still be dx (why?). So we will be using the washer method. The formula is

$$V = \int_4^7 \pi \left[r_O^2 - r_I^2\right] dx,$$

where r_O and r_I are the outside and inside radii respectively. So we need to find the two radii:

$$r_O = 5 - y_1 = 5 - \left(\frac{1}{2}x - 2\right) = 7 - \frac{1}{2}x$$

$$r_I = 5 - y_2 = 5 - \left(-(x-4)\left(x - \frac{15}{2}\right)\right) = 5 + (x-4)\left(x - \frac{15}{2}\right).$$

Our integral becomes

$$V = \int_4^7 \pi \left[\left(7 - \frac{1}{2}x\right)^2 - \left(5 + (x-4)\left(x - \frac{15}{2}\right)\right)^2\right] dx = \frac{603}{20}\pi.$$