

Math 2315 - Calculus II

Quiz #7 - 2007.10.01

Solutions

Determine whether the following improper integral converges and, if so, evaluate it.

$$\int_1^2 \frac{1}{x \ln(x)} dx$$

So we have a discontinuity at $x = 1$. So we rewrite:

$$\int_1^2 \frac{1}{x \ln(x)} dx = \lim_{t \rightarrow 1^+} \int_t^2 \frac{1}{x \ln(x)} dx$$

We can perform a substitution by setting $u = \ln(x)$. The resulting integral gives an obvious solution. After rewriting back in terms of x we get

$$\lim_{t \rightarrow 1^+} \int_t^2 \frac{1}{x \ln(x)} dx = \lim_{t \rightarrow 1^+} [\ln(\ln(x))] \Big|_t^2.$$

So we can evaluate at the top and bottom as follows:

$$\begin{aligned} \ln(\ln(2)) - \lim_{t \rightarrow 1^+} \ln(\ln(t)) &= \ln(\ln(2)) - \ln\left(\lim_{t \rightarrow 1^+} \ln(t)\right) \\ &= \ln(\ln(2)) - \ln\left(\lim_{t \rightarrow 0^+}\right) \\ &= \ln(\ln(2)) + \infty \\ &= \infty. \end{aligned}$$