Math 2315 - Calculus II

Final Exam - 2010.04.26

Due Date - 2010.05.10 at 11:00 AM

Name: ______________________

Instructions:

Please show all of your work, allow plenty of space between each problem, or part of problem; however, you do not have to start each problem on a separate sheet of paper. The only ways that you can get help on any of these problems is to look through books, look through notes, look at old course material on my website, or talk to me. Nothing else is allowed. Do not use decimal approximations for any numbers, and all problems in this exam do not require the use of a calculator, but you can use your calculator to verify your answers.

Problem Point Distribution:

1 a)	5pts	5	10pts	14	7pts
1 b)	5pts	6 a)	6pts	15 a)	4pts
1 c)	5pts	6 b)	6pts	15 b)	5pts
1 d)	5pts	6 c)	6pts	15 c)	6pts
1 e)	5pts	7	8pts	15 d)	5pts
2 a)	5pts	8	10pts	16	10pts
2 b)	5pts	9	10pts	17	15pts
3 a)	5pts	10 a)	7pts		
3 b)	5pts	10 b)	7pts		
3 c)	5pts	10 c)	7pts		
3 d)	5pts	11	8pts		
4 a)	5pts	12	6pts		
4 b)	5pts	13 a)	3pts		
4 c)	5pts	13 b)	5pts		
4 d)	5pts	13 c)	6pts	Total	232pts

1. Compute the following integrals.

a) $$\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$$ b) $$\int \frac{4-w}{w(w^2+2)^2}\,dw$$

c) $$\int \frac{z}{z^{1/2}-z^{1/3}}\,dz$$ d) $$\int \frac{\ln(\ln(x))\ln(x)}{x}\,dx$$

e) $$\int_{-\infty}^{\infty} \frac{1}{\cosh^2(t)+\sinh^2(t)}\,dt$$

2. Determine if the following integrals are convergent or divergent.

a) $$\int_1^{\infty} \frac{1}{e^{-(x+1/x)}}\,dx$$ b) $$\int_0^1 \frac{1}{z^4+\sqrt{z}}\,dz$$

3. Compute the limit as $n\to\infty$ for each of the following sequences, if they exist.

a) $$a_n = \frac{e^n+3^n}{5^n}$$ b) $$b_n = n\sin\left(\frac{\pi}{n}\right)$$

c) $$c_n = \sqrt[3]{n+1}-n$$ d) $$d_n = (2^n+3^n)^{1/n}$$

4. Determine if the following series are convergent or divergent.

a) $$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3+3\ln(n)}$$ b) $$\sum_{n=2}^{\infty} \frac{1}{n^{\ln(n)}}$$

c) $$\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\frac{1}{n}}$$ d) $$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

5. Find the power series representation of $f(x)=\frac{1}{(1-x)^2}$. Use your representation to give an exact value of $\displaystyle\sum_{n=1}^{\infty}\frac{n}{2^n}$

6. Find the values of x, along with the radius of convergence, for which the following power series converge.

a) $$\sum_{n=2}^{\infty} \frac{x^n}{\ln(n)}$$ b) $$\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n^2+3}}$$ c) $$\sum_{n=0}^{\infty} \frac{(-1)^n(2x)^n}{\sqrt{n^2+3}}$$

7. Find the first four nonzero terms in the Maclaurin series for the function $f(x)=e^x\ln(1-x)$.

8. Find the solution to the following initial value problem in explicit form.

$$y^2(1-x^2)^{\frac{1}{2}}\frac{dy}{dx} = \sin^{-1}(x),\ \ y(0)=0.$$

9. Use a series representation of $\sin(3x)$ to find values of r and s for which

$$\lim_{x\to 0}\left(\frac{\sin(3x)}{x^3}+\frac{r}{x^2}+s\right)=0.$$

10. Compute the following limits:

a) $\displaystyle\lim_{z\to 0}\frac{1-\cos^2(z)}{\ln(1-z)+\sin(z)}$ b) $\displaystyle\lim_{x\to\infty}\int_x^{2x}\frac{1}{t}\,dt$ c) $\displaystyle\lim_{x\to\frac{\pi}{2}}\frac{\ln(\csc(x))}{(x-\frac{\pi}{2})^2}$

11. Find a value of c that makes the piecewise function

$$g(\theta)=\begin{cases}\dfrac{\tan^2(\theta)}{\sin\left(\frac{4\theta^2}{\pi}\right)}, & \theta\neq 0\\ c, & \theta=0\end{cases}$$

continuous from the right at $\theta=0$. Explain why your value of c works.

12. Show that $r=a\cos(\theta)+b\sin(\theta)$ is the equation of a circle passing through the origin. Express the radius and center (in rectangular coordinates) in terms of a and b.

13. Consider the polar equation $r^2=\cos(2\theta)$.

a) Give a range of values for θ which gives the entire graph to the equation.

b) Plot this equation for the given range of θ from part a).

c) Find the area of one loop of the graph given in part b).

14. Find the length of the cardiod $r=1+\cos(\theta)$

15. Consider the parametric curve defined by $(x(t),y(t))=(t^2+1,t^3-4t)$.

a) Graph the curve for $-3\leq t\leq 3$.

b) Find the coordinates of the points, and the corresponding times, where the curve has horizontal or vertical tangent lines.

c) Find the area of the enclosed enclosed region defined by the parametric curve.

d) Find the two values of t, t_+ and t_-, such that the tangent line to the parametric curve at time t_+ and t_- passes through the origin. *You will have to use your calculator for this one, the values of* t_+ *and* t_- *are not pretty.*

16. Consider the line L given by $y=-x+1$, and the point $P=P(a,b)$. Prove that the shortest distance between the point P and the line L is

$$d(P,L)=\sqrt{\frac{1}{2}(1-(a+b))^2}.$$

17. Derive the equation of the parabola defined by the set of points equidistant from the line L given by $y=-x+1$, and the point $P=P(1,1)$. Sketch your result as well. Your final answer should be equivalent to

$$y^2+x^2-2(xy+x+y)+3=0.$$

Hint: The previous problem makes this problem much easier!