Math 2315 - Calculus II

Homework #11 Solutions

Assigned - 2010.02.22

Due - 2010.03.01

Textbook problems:

Section 9.1 - 1-4 all, 12, 15, 17, 20, 22, 24, 27, 28, 29, 32, 34, 37, 38, 29, 40, 41

Fun Problems:

1. If $a_n = \sqrt{n+3} - \sqrt{n}$, compute $\lim_{n\to\infty} a_n$.

$$\begin{aligned}
\lim_{n\to\infty} a_n &= \lim_{n\to\infty} \sqrt{n+3} - \sqrt{n} \\
&= \lim_{n\to\infty} \sqrt{n+3} - \sqrt{n} \cdot \frac{\sqrt{n+3}+\sqrt{n}}{\sqrt{n+3}+\sqrt{n}} \\
&= \lim_{n\to\infty} \frac{n+3-n}{\sqrt{n+3}+\sqrt{n}} \\
&= \lim_{n\to\infty} \frac{3}{\sqrt{n+3}+\sqrt{n}} \\
&= 0.
\end{aligned}$$

2. If $b_n = n^2\left(\sqrt[3]{n^3+1} - n\right)$, compute $\lim_{n\to\infty} b_n$.

$$\begin{aligned}
\lim_{n\to\infty} b_n &= n^2\left(\sqrt[3]{n^3+1} - n\right) \\
&= \lim_{n\to\infty} n^2\sqrt[3]{n^3+1} - n^3 \\
&= \lim_{n\to\infty} \frac{\frac{1}{n}\sqrt[3]{n^3+1} - 1}{\frac{1}{n^3}} \\
&= \lim_{n\to\infty} \frac{\sqrt[3]{1+\frac{1}{n^3}} - 1}{\frac{1}{n^3}}
\end{aligned}$$

This is now in the form $\frac{0}{0}$ so we apply l'Hospital's rule to get

$$\begin{aligned}
\lim_{n\to\infty} \frac{\sqrt[3]{1+\frac{1}{n^3}} - 1}{\frac{1}{n^3}} &= \lim_{n\to\infty} \frac{\frac{1}{3}\left(1+\frac{1}{n^3}\right)^{-\frac{2}{3}}\left(-3n^{-4}\right)}{-3n^{-4}} \\
&= \frac{1}{3}.
\end{aligned}$$

3. Let $c_n = \frac{\sqrt[n]{n!}}{n}$. Show that $\ln(c_n) = \frac{\ln(n!) - n\ln(n)}{n}$.

We show this as follows:

$$\begin{aligned}\ln\left(\frac{\sqrt[n]{n!}}{n}\right) &= \ln\left((n!)^{\frac{1}{n}}\right) - \ln(n) \\ &= \frac{1}{n}\ln(n!) - \ln(n) \\ &= \frac{\ln(n!) - n\ln(n)}{n}.\end{aligned}$$