Math 2315 - Calculus II

Homework #2 Solutions

Assigned - 2010.01.20

Due - 2010.01.25

Textbook problems:

Section 6.2 - 1, 4, 19, 22, 37, 38, 43

Section 6.3 - 1, 2, 5, 7, 10, 13, 14, 19, 24, 27, 30, 34, 35, 38, 51, 54, 56, 57, 60

Fun Problems:

1. Show that if $f(x)$ is odd and $f^{-1}(x)$ exists, then $f^{-1}(x)$ is odd.

We simply need to show that $f^{-1}(-x) = -f^{-1}(x)$. Given that x, and $-x$ are in the domain of $f(x)$, we set $y = f(x)$. Hence, $-y = f(-x)$. Applying f^{-1} to both of these gives $f^{-1}(y) = x$ and $f^{-1}(-y) = -x$.

2. Argue that an even function cannot have an inverse on an interval which contains the origin.

We simply assume that $f(x)$ has an inverse function. If $f(x)$ is even about the origin, then $f(-a) = y$ and $f(a) = y$, hence $f^{-1}(y) = \pm a$, which contradicts the fact that f^{-1} is a function.

3. Prove that if $\lambda < 0$, the equation $e^x = \lambda\, x$ has a unique solution.

Consider the function $F(x) = e^x - \lambda\, x$. If $\lambda < 0$, set $\alpha = -\lambda$, for $\alpha > 0$, then $F(x) = e^x + \alpha\, x$. Notice that $F'(x) = e^x + \alpha > 0$ for all $x \in \mathbb{R}$, hence $F(x)$ is a strictly increasing function. Next we compute the limits as $x \to \pm\infty$:

$$\lim_{x\to-\infty} F(x) = \lim_{x\to-\infty} e^x + \alpha \lim_{x\to-\infty} x = -\infty,$$

$$\lim_{x\to\infty} F(x) = \lim_{x\to\infty} e^x + \alpha \lim_{x\to\infty} x = \infty.$$

With the above two limits, we can invoke the Intermediate Value Theorem to prove that there exists at least one root of $F(x)$, hence a solution to $e^x = \lambda\, x$. The root is unique since $F(x)$ is strictly increasing.