Math 2315 - Calculus II

Homework #4 Solutions

Assigned - 2010.01.26

Due - 2010.02.01

Textbook problems:

Section 6.6 - 1, 4, 6, 7, 12, 16, 18, 21, 26, 32, 41, 42, 43

Fun Problems:

1. Prove that for $n \geq 2$

$$\frac{d}{dx}\left[\frac{1}{n}\cosh^{n-1}(x)\sinh(x)\right] = \cosh^n(x) - \frac{n-1}{n}\cosh^{n-2}(x).$$

$$\begin{aligned}
\frac{d}{dx}\left[\frac{1}{n}\cosh^{n-1}(x)\sinh(x)\right] &= \frac{1}{n}\cosh^{n-1}(x)\cosh(x) + \frac{n-1}{n}\cosh^{n-2}(x)\sinh^2(x) \\
&= \frac{1}{n}\cosh^n(x) + \frac{n-1}{n}\cosh^{n-2}(x)\sinh^2(x) \\
&= \frac{1}{n}\cosh^n(x) + \frac{n-1}{n}\cosh^{n-2}(x)\left(\cosh^2(x) - 1\right) \\
&= \frac{1}{n}\cosh^n(x) + \frac{n-1}{n}\cosh^n(x) - \frac{n-1}{n}\cosh^{n-2}(x) \\
&= \frac{1+n-1}{n}\cosh^n(x) - \frac{n-1}{n}\cosh^{n-2}(x) \\
&= \frac{n}{n}\cosh^n(x) - \frac{n-1}{n}\cosh^{n-2}(x) \\
&= \cosh^n(x) - \frac{n-1}{n}\cosh^{n-2}(x)
\end{aligned}$$

2. Prove that for $n \geq 2$

$$\int \cosh^n(x)\,dx = \frac{1}{n}\cosh^{n-1}(x)\sinh(x) + \frac{n-1}{n}\int \cosh^{n-2}(x)\,dx.$$

Using problem 1, we have that

$$\frac{1}{n}\cosh^{n-1}(x)\sinh(x) = \int \cosh^n(x) - \frac{n-1}{n}\cosh^{n-2}(x)dx.$$

With some algebraic manipulation, we get:

$$\int \cosh^n(x)\,dx - \frac{n-1}{n}\int \cosh^{n-2}(x)\,dx = \frac{1}{n}\cosh^{n-1}(x)\sinh(x),$$

and solving for the first integral gives

$$\int \cosh^n(x)\,dx = \frac{1}{n}\cosh^{n-1}(x)\sinh(x) + \frac{n-1}{n}\int \cosh^{n-2}(x)\,dx.$$

3. Evaluate the following integral:

$$\int \frac{\tanh^{-1}(x)}{x^2 - 1}\, dx$$

Letting $u = \tanh^{-1}(x)$, $du = \dfrac{1}{1 - x^2} dx$, so

$$\begin{aligned}
\int \frac{\tanh^{-1}(x)}{x^2 - 1}\, dx &= \int -u\, du \\
&= -\frac{1}{2}u^2 + c \\
&= -\frac{1}{2}\tanh^{-2}(x) + C
\end{aligned}$$