Math 2315 - Calculus II

Quiz #5 - 2010.02.22

Solutions

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1. Compute the following Integral:

$$\int_{-\infty}^{2} (x^2+1)\, e^{2x-1}\, dx$$

First we break up the integral:

$$\int_{-\infty}^{2} (x^2+1)\, e^{2x-1}\, dx = \lim_{R\to-\infty} \int_{R}^{2} x^2\, e^{2x-1}\, dx + \lim_{R\to-\infty} \int_{R}^{2} e^{2x-1}\, dx$$

and then we apply integration by parts (using tabular integration) to the first integral:

$$\begin{aligned}\lim_{R\to-\infty} \int_{R}^{2} x^2\, e^{2x-1}\, dx &= \lim_{R\to-\infty} \frac{1}{2}\, x^2\, e^{2x-1} - \frac{1}{2}\, x\, e^{2x-1} + \frac{1}{4}\, e^{2x-1}\Big|_{R}^{2} \\ &= \frac{5}{4}\, e^3 - 0.\end{aligned}$$

The second term is a more simple integral, which we had to find to perform integration by parts on the first integral.

$$\begin{aligned}\lim_{R\to-\infty} \int_{R}^{2} e^{2x-1}\, dx &= \frac{1}{2}\, e^{2x-1}\Big|_{R}^{2} \\ &= \frac{1}{2}\, e^3 - 0\end{aligned}$$

Putting these two integrals together gives:

$$\int_{-\infty}^{2} (x^2+1)\, e^{2x-1}\, dx = \frac{5}{4}\, e^3 + \frac{1}{2}\, e^2 = \frac{7}{4}\, e^3.$$