Math 2315 - Calculus II

Quiz #7 - 2010.03.10

Solutions

Determine convergence or divergence of the following series:

1. $\sum_{k=-2}^{\infty} 4\left(\frac{8}{9}\right)^k$

This is a geometric series (removing the $k = -2, -1, 0$ terms, all of which are finite), with $r = \frac{8}{9} < 1$. Thus the series is convergent.

2. $\sum_{k=6}^{\infty} \frac{2k + 3\cos(2k+1)}{k^3}$

This can be done several ways, but we will use the comparison test. We start with the following observation for all k:

$$-3 \leq 3\cos(2k+1) \leq 3,$$

and therefore,

$$-3 + 2k \leq 2k + 3\cos(2k+1) \leq 3 + 2k,$$

which gives (at least for $k > 6$ which we are concerned with):

$$0 \leq \frac{2k + 3\cos(2k+1)}{k^3} \leq \frac{3 + 2k}{k^3}.$$

We now have

$$\sum_{k=6}^{\infty} \frac{2k + 3\cos(2k+1)}{k^3} \leq \sum_{k=6}^{\infty} \frac{2k+3}{k^3}.$$

We can show that

$$\sum_{k=6}^{\infty} \frac{2k+3}{k^3} < \infty$$

by using the limit comparison test with $a_k = \frac{2k+3}{k^3}$ and $b_k = \frac{1}{k^2}$. Therefore, the series is convergent!