

Math 2315 - Calculus II
Quiz #8 - 2010.03.30
Solutions

Find the first four terms in the Taylor series of $\sin(x)$ centered at $c = \frac{\pi}{4}$.

So first we write out the formula, given $f(x) = \sin(x)$:

$$\sin(x) \approx f\left(\frac{\pi}{4}\right) + f'\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right) + \frac{1}{2!}f''\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)^2 + \frac{1}{3!}f'''\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)^3$$

All that is left to do now is fill in the function values. We have

$$\begin{aligned}f\left(\frac{\pi}{4}\right) &= \frac{1}{\sqrt{2}} \\f'\left(\frac{\pi}{4}\right) &= \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \\f''\left(\frac{\pi}{4}\right) &= -\sin\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} \\f'''\left(\frac{\pi}{4}\right) &= -\cos\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}},\end{aligned}$$

which allow us to complete the first four terms of the Taylor series of $\sin(x)$ centered at $c = \frac{\pi}{4}$:

$$\sin(x) \approx \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right) - \frac{1}{2!}\frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right)^2 - \frac{1}{3!}\frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right)^3$$