Math 3113 - Multivariable Calculus

Homework #1 - 2008.01.10
Due Date - 2008.01.18
Solutions

1. Find an equation of the largest sphere with center $(3, 7, 2)$ that is contained in the first octant.

Since the height from the xy-plane is the shortest, at a value of 2, this is our radius. The equation of the sphere is

$$(x-3)^2 + (y-7)^2 + (z-2)^2 = 2^2.$$

2. Find the point on the sphere $(x-2)^2 + (y-4)^2 + (z+6)^2 = 1$ which is closest to the xy-plane.

The sphere has a center of $(2, 4, -6)$ and a radius of 1. Since the point on the sphere closest to the xy-plane has to be the point with the z value closest to $z = 0$, which occurs at the top of the sphere, the point in question is $(2, 4, -5)$.

3. Find an equation of a cylinder if one point on the cylinder is given by $(2, 2, 2)$ and a point exactly on the opposite side of the first is given by $(4, 2, 12)$.

We only need the first two coordinates of each point for this problem. They are $(2, 2)$ and $(4, 2)$. So the cylinder is centered at $(3, 2)$ and has a radius of 1. The equation of the cylinder is

$$(x-3)^2 + (y-2)^2 = 1.$$

4. Describe what the surface in $\mathbb{R}^3$ represented by the equation $x^2 + (y-x)^2 = 1$ looks like.

The curve $x^2 + (y-x)^2 = 1$ is an elongated ellipse centered at the origin in the xy-plane, so the surface is simply an elongated cylinder in $\mathbb{R}^3$.

5. Find an equation of the plane parallel to the xz-plane which passes through the point $(3, 6, -4)$.

Since the plane is parallel to the xz-plane, the equation of the plane is simply $y = 6$.

6. Let A, B, C and D be the four corners of a quadrilateral. Compute $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA}$.

The vectors simply trace the outside of the quadrilateral starting at point A, then going through B, then C and D before making it back to A. Therefore, the sum must be zero.

7. Let $\overrightarrow{v} \in \mathbb{R}^3$ be a position vector. Prove that $|\overrightarrow{v}| = 1$ if and only if the terminal point of $\overrightarrow{v}$ lies on the unit sphere.

We can let $\overrightarrow{v} = \langle x, y, z \rangle$ for arbitrary values x, y and z.
If $|\overrightarrow{v}| = 1$, then by definition, $|\overrightarrow{v}| = \sqrt{x^2 + y^2 + z^2}$. Setting $\sqrt{x^2 + y^2 + z^2} = 1$ gives $x^2 + y^2 + z^2 = 1$. Thus if $|\overrightarrow{v}| = 1$, the tip of $\overrightarrow{v}$ lies on the unit sphere.
Now the other way. If the terminal point of $\overrightarrow{v}$ lies on the unit sphere, then $x^2 + y^2 + z^2 = 1$ and thus $\sqrt{x^2 + y^2 + z^2} = 1$ and hence $|\overrightarrow{v}| = 1$.