Math 3113 - Multivariable Calculus

Quiz #6 - 2008.03.07

Solutions

Given $F(x,y) = x^3y^2 + \sin(x+y)$ with $x(s,t) = e^{2t+3s}$ and $y(s,t) = st^3 - \frac{t}{s}$,

1. Compute $\frac{\partial F}{\partial t}$

First we apply the chain rule:

$$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial t},$$

and then we compute the pieces:

$$\frac{\partial F}{\partial x} = 3x^2y^2 + \cos(x+y), \quad \frac{\partial F}{\partial y} = 2x^3y + \cos(x+y)$$

$$\frac{\partial x}{\partial t} = 2e^{2t+3s}, \quad \frac{\partial y}{\partial t} = 3t^2s - \frac{1}{s}.$$

This gives

$$\frac{\partial F}{\partial t} = \left(3x^2y^2 + \cos(x+y)\right)\left(2e^{2t+3s}\right) + \left(2x^3y + \cos(x+y)\right)\left(3t^2s - \frac{1}{s}\right),$$

and now we plug in what x and y truly are to get

$$\begin{aligned}\frac{\partial F}{\partial t} = &\left(3\left(e^{2t+3s}\right)^2\left(st^3 - \frac{t}{s}\right)^2 + \cos\left(e^{2t+3s} + st^3 - \frac{t}{s}\right)\right)\left(2e^{2t+3s}\right) \\ &+ \left(2\left(e^{2t+3s}\right)^3\left(st^3 - \frac{t}{s}\right) + \cos\left(e^{2t+3s} + st^3 - \frac{t}{s}\right)\right)\left(3t^2s - \frac{1}{s}\right).\end{aligned}$$

2. Compute $\frac{\partial F}{\partial s}$

First we apply the chain rule:

$$\frac{\partial F}{\partial s} = \frac{\partial F}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial s},$$

and notice we have already compute two of the four partial derivatives. So we only have to compute the following two:

$$\frac{\partial x}{\partial s} = 3e^{2t+3s}, \quad \frac{\partial y}{\partial t} = t^3 + \frac{t}{s^2}.$$

This gives

$$\frac{\partial F}{\partial s} = \left(3x^2y^2 + \cos(x+y)\right)\left(3e^{2t+3s}\right) + \left(2x^3y + \cos(x+y)\right)\left(t^3 + \frac{t}{s^2}\right),$$

and now we plug in what x and y truly are to get

$$\begin{aligned}\frac{\partial F}{\partial s} = &\left(3\left(e^{2t+3s}\right)^2\left(st^3 - \frac{t}{s}\right)^2 + \cos\left(e^{2t+3s} + st^3 - \frac{t}{s}\right)\right)\left(3e^{2t+3s}\right) \\ &+ \left(2\left(e^{2t+3s}\right)^3\left(st^3 - \frac{t}{s}\right) + \cos\left(e^{2t+3s} + st^3 - \frac{t}{s}\right)\right)\left(t^3 + \frac{t}{s^2}\right).\end{aligned}$$