

# Math 3283 - Foundations of Math

Exam #1 - 2010.02.12

Name: \_\_\_\_\_

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1. Determine if each of the following sentences are statements.

a)  $2 + 3 = 6$ .

b) Differentiate  $f(x) = \sin(x)$ .

c) This book has many errors.

d) This book has 14 chapters.

2. Write each of the following statements in symbolic form, where we define the following simple statements symbolically by:

p:  $2 + 3 = 5$ , q:  $7x = 3$ , r:  $f(x) = \sin(x)$  and s:  $g(x) = \tan^{-1}(x)$

a) If  $2 + 3 \neq 5$  or  $7x = 3$  then  $f(x) = \sin(x)$  or  $g(x) = \tan^{-1}(x)$

b) If  $2 + 3 = 5$  or  $7x = 3$  then  $f(x) \neq \sin(x)$ , or  $g(x) = \tan^{-1}(x)$

c)  $g(x) = \tan^{-1}(x)$  iff  $f(x) = \sin(x)$  and  $7x = 3$ , then  $2 + 3 \neq 5$ .

3. Construct truth tables for each of the following statements.

a)  $\sim p \vee q \rightarrow (p \rightarrow q)$

b)  $(p \wedge q \rightarrow r) \rightarrow (p \rightarrow r)$

4. State the a) inverse, b) converse and c) contrapositive of the following conditional statement:

If  $x < 3$  then  $y = 5$  or  $z \geq 2$ .

5. Given that the following arguments are logically valid:

a) 
$$\frac{p \rightarrow q}{\sim q} \quad \therefore \sim p$$

b) 
$$\frac{p \rightarrow q}{p} \quad \therefore q$$

c) 
$$\frac{p \vee q}{\sim q} \quad \therefore p$$

Verify that the following argument is valid without using truth tables:

$$\frac{p \wedge \sim r \rightarrow \sim p \vee q}{p \wedge \sim q} \quad \therefore r$$

6. Let  $p(x, y) : y = x(x - 1)(x + 1)$ , for  $x, y \in \mathbb{R}$ . Determine if the following propositions are true or false.

a)  $\exists! x p(x, 0)$

b)  $\forall y \exists x p(x, y)$

c)  $\forall x \exists y p(x, y)$

d)  $\exists x \forall y p(x, y)$

e)  $\exists y \forall x p(x, y)$

f)  $\forall y \exists! x p(x, y)$

g)  $\forall x \exists! y p(x, y)$

7. Negate the following statements.

a)  $\forall x \exists y p(x, y) \vee q(x)$

b)  $\exists x \forall y \sim p(x, y) \wedge q(x)$

c)  $\forall x \forall y p(x, y) \rightarrow q(x)$