Math 3283 - Foundations of Math

Exam #2 - 2010.03.24

Name: ______________________________

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1. Prove or disprove: For all integers a, b and c, if $a \big| b$ and $b \big| c$, then $a \big| (b + c)$.

2. Prove: nm is even if and only if at least one of n and m is even.

3. Prove or disprove: For all $x \in \mathbb{R}$, if $x > 0$, then $x < x^2$.

4. Prove or disprove: $\exists\, x \in \mathbb{N}$ such that $x + x = x^2$.

5. State which method of proof should be used on the following statement:

$$(p_1 \vee p_2 \vee p_3) \longrightarrow (q_1 \wedge q_2)$$

Also, explain how your method would be used to prove this statement.

The Gamma function is defined as:

$$\Gamma(z) = \int_0^\infty t^{z-1}\, e^{-t}\, dt,$$

and has many interesting properties. One of the most interesting of these properties is that

$$\Gamma(n) = n!, \;\; n \in \mathbb{N}.$$

We will prove this fact by induction, using the following steps.

A. Show that $\Gamma(1) = 1$, that is:

$$\Gamma(1) = \int_0^\infty t^0\, e^{-t}\, dt = 1.$$

B. Show that $\Gamma(n+1) = n\,\Gamma(n)$ for $n \in \mathbb{N}$, that is:

$$\int_0^\infty t^n\, e^{-t}\, dt = \int_0^\infty n\, t^{n-1}\, e^{-t}\, dt.$$

Hint: Remember $\int_0^\infty u'\, v\, dt = u\, v\Big|_0^\infty - \int_0^\infty u\, v'\, dt$, so let $u' = n\, t^{n-1}$ and $v = e^{-t}$ on the right hand side of the above equation.

C. Using induction and parts A and B, prove that $\Gamma(n) = n!$, for $n \in \mathbb{N}$.