Math 3283 - Foundations of Math

Final Exam - 2010.05.12

Name: ____________________________

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1. Consider a statement of the form

$$(p_1 \vee p_2) \rightarrow (q_1 \wedge q_2)\,,$$

for p_1, p_2, q_1 and q_2 simple logical statements.

a) If you wish to directly prove a statement of this form, what do you assume?

b) If you wish to prove this statement by contraposition, what do you assume?

c) If you wish to prove this statement by contradiction, what do you assume?

d) Write the converse of the given statement.

2. Construct a truth table for the statement

$$(q\wedge \sim p) \rightarrow r \vee p \leftrightarrow p$$

3. Write the negation of the following quantified statement *without using the symbol* '→':

$$\forall a, b \in I, \exists c \in (a,b)\,[f(a) = f(b) \rightarrow f'(c) = 0]\,.$$

4. Let $U = \{-1, 0, 1, 2, 3\}$, $V = \{-3, -2, -1, 0, 1, 2\}$. Determine whether the following quantified statements are true or false. If the statement is false, give a counterexample to prove your point. In either case, explain your answer fully.

a) $\forall x \in U, \exists y \in V, (x + y = 0)$.

b) $\forall x \in U, \exists! y \in V, (x + y = 0)$.

c) $\forall x \in V, \exists y \in U, (x + y = 0)$.

d) $\forall x \in V, \forall y \in U, (x + y = 0)$.

e) $\exists x \in U, \exists y \in V, (x + y = 0)$.

f) $\exists y \in U, \forall x \in V, (x + y = 0)$.

5. Let $m, n \in \mathbb{Z}$. Prove that if n is odd and $n + m$ is even, then m is odd.

6. Prove that for every positive integer n, $3 \,|\, 2n^3 + 4n + 9$.

7. Prove, or disprove, that an integer n is odd if and only if $6n - 1$ is odd.

8. Let $A = \{0, 1\}$, compute $\mathcal{P}\left(\mathcal{P}(A)\right)$.

9. Construct a relation, R, on the set $A = \{-1, 0, 1, a\}$, which includes the points $(-1, a)$, $(a, 1)$, and $(a, -1)$, and is also transitive.

10. Let $A = \{0, 1, 2\}$ and $B = \{a, b\}$.

a) Compute $A \times B$.

b) Compute $B \times A$.

c) What is $|A \times B|$?

11. Compute, and give arguments for your reasoning, the following infinite unions and intersections.

a) $\displaystyle\bigcap_{n=1}^{\infty}\left[2+\frac{1}{n},4-\frac{1}{n}\right)$

b) $\displaystyle\bigcup_{n=1}^{\infty}\left(2+\frac{1}{n},4-\frac{1}{n}\right]$

12. Prove that if $f : A \to B$ is onto, and $G : B \to C$ is onto, then $g \circ f : A \to C$ is onto.

13. Prove that if $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.