Physics 1214 — Chapter 18: Electric Potential and Capacitance

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1 Electric Potential Energy

Essential Points from Chapter 7:

- when a constant force $\vec{F}$ acts on a particle that moves in a straight line through a displacement $\vec{s}$ from point a to point b, the work $W_{a\to b}$ done by the force is

$$W_{a\to b} = Fs\cos\phi$$

  where ϕ is the angle between the force and displacement. Let x point in the direction of the particle's motion.

- because the force field is *conservative*, the work that is done can always be expressed in terms of a *potential energy U*. When the particle moves from a point where the potential energy is U_a to a point where it is U_b, the work $W_{a\to b}$ done by the force is

$$W_{a\to b} = U_a - U_b.$$

- the work-energy theorem says that the change in kinetic energy $\Delta K = K_b - K_a$ during any displacement is equal to the total work done on the particle so if $W_{a\to b} = U_a - U_b$ is the *total* work, then $K_b - K_a = U_a - U_b$ which is usually written as

$$K_a + U_a = K_b + U_b.$$

Applied to electrical forces, a uniform electric field with magnitude, E, exerts a *constant force* on a positive test charge, q', of $F = q'E$ (for a positive test charge in a positive field) then for a distance, s:

$$W_{a\to b} = Fs = q'Es.$$

Potential Energy of Point Charges

For work in a non-constant field, such as the work, W, done on a test charge, q', when it moves in an electric field caused by a single stationary point charge, q. (See graph on p. 585)

$$W_{a\to b} = kqq'\left(\frac{1}{a} - \frac{1}{b}\right)$$

and from this, ***potential energy***

$$U_a = \frac{kqq'}{a} \text{ and } U_b = \frac{kqq'}{b}.$$

It can be shown that the work $W_{a\to b}$ done on q' by the $\vec{\mathbf{E}}$ field produced by q is the same for *all possible paths* from a to b. And the *total work* done in a roundtrip displacement (from a back to a) is zero.
These are the characteristics of a *conservative force.*

Potential energy of point charges

The potential energy U of s system consisting of a point charge q' located in the field produced by a stationary point charge q, at a distance r from the charge, is

$$U = k\frac{qq'}{r}.$$

Note: As the distance, r, goes to infinity, U goes to zero.

For a collection of charges,

$$U = kq' \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + ... \right).$$

Every electric field due to a static charge distribution is a conservative force field.

2 Potential

Electric Potential or Potential

The *electric potential* V at any point in an *electric field* is the *electric potential energy* U per unit charge associated with a test charge q' at that point:

$$V = \frac{U}{q'}$$

or

$$U = q'V.$$

Potential energy and charge are both scalars, so potential is a scalar quantity .
Units: The SI unit of potential, 1 J/C, is called one ***volt*** (1V).

1 V = 1 volt = 1 J/C = 1 joule/coulomb

voltage: the electric potential in electric circuits.
potential difference: the difference in electric potential between two points in a system.

In a "work per unit charge" basis,

$$\frac{W_{a\to b}}{q'} = \frac{U_a}{q'} - \frac{U_b}{q'} = V_a - V_b$$

Potential of a point charge

When a test charge q' is a distance r from a point charge q, the potential V is

$$V = \frac{U}{q'} = k\frac{q}{r},$$

where k is the same constant as in Coulomb's law.

To find the potential V at a point due to any collection of point charges:

$$V = \frac{U}{q'} = k \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + ... \right)$$

$$1\text{V} = 1\frac{\text{J}}{\text{C}} = 1\frac{(1\text{N})(1\text{m})}{1\text{C}} \text{ and } 1\text{N/C} = 1\text{V/M}$$

3 Equipotential Surfaces

equipotential surface: a surface on which the *potential* is the same at every point.

- where there exists an $\vec{\mathbf{E}}$, we can construct an *equipotential surface* through any point
- in diagrams, there is usually only a few representative *equipotentials* shown
- no point can be at two different potentials, so *equipotential surfaces* for different potentials can never touch or intersect
- 3-D surfaces cannot be drawn on a 2-D diagram so we draw lines representing the intersections of *equipotential surfaces* with the plane of the diagram

Field lines and *equipotential surfaces* are always mutually perpendicular.

For electrostatic charges, the electric field, $\vec{\mathbf{E}}$, just outside a conductor must be perpendicular to the surface at every point.

For an electrostatic system, a conducting surface is always an *equipotential surface.*

Electric field represented as potential gradient

The magnitude of the electric field at any point on an *equipotential surface* equals the rate of change of potential, ΔV, with distance Δs as the point moves perpendicularly from the surface to an adjacent one a distance Δs away:

$$E = -\frac{\Delta V}{\Delta s}.$$

The negative sign shows that when a point moves in the direction of the electric field, the potential decreases. The quantity $\Delta V/\Delta s$, representing a rate of change of V with distance, is called the ***potential gradient***; this is an alternative name for electric field.

$$E_x = -\frac{\Delta V}{\Delta x}, \quad E_y = -\frac{\Delta V}{\Delta y}, \quad E_z = -\frac{\Delta V}{\Delta z}$$

4 The Millikan Oil-Drop Experiment

- two parallel, horizontal metal plates, insulated and separated, maintained at a *potential difference*
- oil drops are sprayed from an atomizer and acquire a charge
- a few drops fall from a hole in the top plate and are observed with a telescope equipped with a scale allowing speed of the drops to be measured
- suppose a drop has a negative charge q
- with a downward electric field magnitude E (between the plates)
- the forces on the oil drop are the downward force $F = mg$ and the upward force $F = qE$
- Millikan adjusted E such that $mg = qE$ and the drop was in static equilibrium (floating!!!) thus $q = mg/E$
- $E = V_{ab}/d$ where d is the distance between the plates
- the mass of the oil drop can be found since the density, ρ, is mass per unit volume $\rho = m/\text{Volume}$
- the oil drop when in static equilibrium is a sphere due to surface tension (think of a drop of water floating in space) (volume of a sphere $= \frac{4}{3}\pi r^3$)

$$m = \frac{4}{3}\pi r^3 \rho, \quad E = \frac{V_{ab}}{d} \text{ thus } q = \frac{4}{3}\frac{\rho\pi r^3 g d}{V_{ab}}$$

The radius of the oil drop was too small to measure! Millikan found the radius by turning off $\vec{\mathbf{E}}$, and measuring the terminal speed (aka terminal velocity), ν_t. **After thousands of drops, every drop had an integer value of e.** See notes from Chapter 17 for the value of e.

electron volt (eV): derived from the change in potential energy, $\Delta U = q(V_b - V_a) = qV_{ba}$. If $V_{ba} = 1$ V, then $\Delta U = (1.602 \times 10^{-19}\text{C})(1\text{V}) = 1.602 \times 10^{-19}\text{J} = 1\text{eV}$.

$$\boxed{1\text{eV} = 1.602 \times 10^{-19}\text{J}}$$

5 Capacitors

capacitor: a device that stores electric potential energy, U, and electric charge.

- in principle, a capacitor consists of any two conductors separated by vacuum or an insulating material
- there is a charge of opposite sign on each conductor, thus an $\vec{\mathbf{E}}$, between the conductors, thus a potential difference, V between them
- in most practical applications, the conductors have charges (Q and $-Q$) with equal magnitude thus net charge is zero

Capacitance

The capacitance, C of a capacitor is the ratio of the magnitude of the charge Q on either conductor to the magnitude of the potential difference V_{ab} between the conductors:

$$C = \frac{Q}{V_{ab}}.$$

Unit: The SI unit of capacitance is 1 farad (1 F).

$$1\text{F} = 1\text{C/V}$$

parallel-plate capacitor: two parallel conducting plates, each with area A, separated by a distance d that is small in comparison with the area.

- nearly all of the field is localized between the plates
- there exists some *fringing* of the field at the edges; generally neglected in our study
- the electric field, $\vec{\mathbf{E}}$, between the plates is uniform

surface charge density (σ): the electric charge per unit area. For a parallel-plate capacitor, the charge densities on the plates are $\sigma = Q/A$ and $\sigma = -Q/A$.

The electric field magnitude:

$$\boxed{E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}}$$

and the potential difference (voltage) between the plates

$$\boxed{V_{ab} = Ed = \frac{1}{\epsilon_0}\frac{Qd}{A}}$$

Capacitance of a parallel-plate capacitor

The capacitance C of a parallel-plate capacitor in vacuum is directly proportional to the area A of each plate and inversely proportional to their separation d:

$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}.$$

6 Capacitors in Series and in Parallel

series connection : two devices connected one after another between points a and b and a constant potential difference V_{ab} is maintained. *The total potential difference across all of the capacitors is the sum of the individual potential differences.*
parallel connection : two devices connected in parallel between points a and b. The upper plates of the capacitors are connected together to form an equipotential surface, and the lower plates form another. *The potential difference is the same for both capacitors.*

Equivalent capacitance of capacitors in series

When capacitors are connected in *series*, the reciprocal of the equivalent capacitance of a series combination equals the sum of the reciprocals of the individual capacitances:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$

The magnitude of charge is the same on all of the plates of all the capacitors, but the potential differences across individual capacitors are, in general, different.

Equivalent capacitance of capacitors in parallel

When capacitors are connected in *parallel*, the equivalent capacitance of the combination equals the sum of the individual capacitances:

$$C_{eq} = C_1 + C_2 + C_3 + ...$$

7 Electric Field Energy

$$\boxed{\Delta W = \nu \Delta q = \frac{q\Delta q}{C}}$$

energy density (u): energy per unit volume in the space between the plates of a parallel-plate capacitor with plate are A and separation d.

$$u = \text{ energy density } = \frac{\frac{1}{2}CV^2}{Ad}.$$

where capacitance $C = \epsilon_0 A/d$ and the potential difference $V = Ed$. For any capacitor in vacuum and for any field configuration in vacuum,

$$\boxed{u = \frac{1}{2}\epsilon_0 E^2.}$$

A vacuum can have electric fields and therefore energy even though there is no matter.

8 Dielectrics

dielectric: a nonconducting material between the plates of a capacitor. Purpose of dielectric:

- solves the mechanical problem of maintaining two large metal sheets at a very small separation without actual contact
- to prevent ***dielectric breakdown***: a partial ionization that permits conduction through a material that is supposed to be an insulator (like air); a sufficiently strong electric field that can cause an insulator to become a conductor.
- the capacitance of a capacitor of given dimensions is greater when there is a dielectric material between the plates than when there is air or vacuum.

dielectric constant: the ratio of the capacitance to the original capacitance

$$\boxed{K = \frac{C}{C_0}.}$$

If the charge is constant, the potential difference is reduced by a factor of K,

$$\boxed{V = \frac{V_0}{K}.}$$

$$\boxed{E = \frac{E_0}{K}}$$

polarization: redistribution of charge within the molecules of the dielectric material.
dielectric strength: the maximum electric field a material can withstand without the occurrence of breakdown.

9 Molecular Model of Induced Charge

polar molecule: a molecule with excess positive charge concentrated on one side of the molecule and negative charge on the other (an electric dipole). When placed in an electric field, polar molecules tend to partially align with the field.