Math 4133 - Linear Algebra

Midterm - Written Portion

Name: _____

Consider the system of equations for problems 1–6.

$$\begin{aligned} x_1 - 2x_2 + 3x_3 + 4x_4 &= 1 \\ -3x_1 + 4x_2 + 6x_3 - 2x_4 + \frac{5}{2}x_5 &= 5 \\ -5x_1 + 6x_2 + 2x_3 + 3x_4 + 5x_5 &= 11 \end{aligned}$$

1. Without solving, what is the most likely value for the dimension of the solution to the above system.

2. Convert the system into an augmented matrix.

3. Row reduce the matrix from problem 2.

4. Write your solution $(x_1, x_2, x_3, x_4, x_5)$ in terms of the variables x_4 and x_5.

5. Write your solution $(x_1, x_2, x_3, x_4, x_5)$ in terms of the variables x_3 and x_5.

6. Is it possible to express the solution to this system in terms of any two variables? If not, give at least one example of a pair of variables for which the solution can not be written in terms of.

Consider the following matrices for problems 7–14.

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 1 & 0 \\ -2 & 5 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

7. Compute the determinant of A by expansion along the second column.

8. Compute the determinant of A by expansion along the second row.

9. Compute the cofactor matrix C for A.

10. Compute the transpose to the cofactor matrix C from problem 9.

11. Compute the inverse to A using the previous problems.

12. Compute the LU factorization of A.

13. Solve $AX = B$ using the LU factorization from problem 12.

14. Solve $AX = B$ by computing $A^{-1}B$. Verify that this approach yields the same answer as that of problem 13.

Consider the vector $\overrightarrow{t} = \langle 1, -2, 1 \rangle$ for problems 15–18.

15. Find a unit vector $\overrightarrow{u}$ in the direction of $\overrightarrow{t}$.

16. Find two vectors, $\overrightarrow{v}$ and $\overrightarrow{w}$, both of which are unit length and satisfy $\overrightarrow{u} \perp \overrightarrow{v}$, $\overrightarrow{u} \perp \overrightarrow{w}$ and $\overrightarrow{v} \perp \overrightarrow{w}$.

17. Define $B \in \mathbb{R}^{3\times 3}$ to be the matrix whose rows are the vectors $\overrightarrow{u}$, $\overrightarrow{v}$ and $\overrightarrow{w}$ from problems 15 and 16. Perform the matrix multiplication $B\,B^T$.

18. Come up for a good reason as to why you should have known your answer to problem 17.

19. Find the distance between the two parallel planes:

$$-5x_1 + 6x_2 + 2x_3 + 3x_4 + 5x_5 = 15$$
$$-5x_1 + 6x_2 + 2x_3 + 3x_4 + 5x_5 = -3$$

Consider the vectors $\overrightarrow{u} = \langle 1, 1, 1\rangle$ and $\overrightarrow{v} = \langle -1, 1, 0\rangle$ for problems 20–21.

20. Find the equation of the plane in which the vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ lie.

21. Project the vector $\overrightarrow{w} = \langle 4, 5, -4\rangle$ onto the plane from problem 20.