Math 4133 - Linear Algebra
Quiz #6 - 2014.02.12
Solutions

Consider the matrices:

$$A = \begin{bmatrix} 1 & 2 & 2 & -1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ -9 \\ -5 \\ 1 \end{bmatrix}$$

1. Compute the determinant of A.

   Since A is upper triangular we simply take the product of the diagonal entries. Thus $\det(A) = 1$.

2. Solve the equation $AX = B$, where $X = [\ x_1 \quad x_2 \quad x_3 \quad x_4\ ]^T$.

   We solve this by backward substitution. The last row gives $x_4 = 1$. The second to last row gives $x_3 + 4x_4 = -5$. Letting $x_4 = 1$ gives $x_3 = -9$. The second row yields $x_2 + 2x_3 - x_4 = -9$, which after plugging in x_4 and x_3 is $x_2 - 18 - 1 = -9$. Solving for x_2 gives $x_2 = 10$. Lastly, the top row gives $x_1 + 2x_2 + 2x_3 - x_4 = 1$. Plugging in the known values of x_2, x_3, and x_4 into this equation gives $x_1 + 20 - 18 - 1 = 1$. Thus $x_1 = 0$. So our solution is $(x_1, x_2, x_3, x_4) = (1, 10, -9, 0)$.