Math 4133 - Linear Algebra

Quiz #7 - 2014.02.14

Solutions

Consider the matrices:

$$B = \begin{bmatrix} 1/6 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ -4 & 2 & 1 & 0 \\ 3 & 1 & 1 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$E = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

If the matrix $A \in \mathbb{R}^{4\times 4}$ (not given above) is invertible, then we know $\det(A) = a \neq 0$. Express each of the following determinants in terms of a.

1. $\det(AB) = a$ since $\det(B) = 1$.

2. $\det(AC) = -a$ since the matrix C corresponds to an elementary where two rows are swapped and the determinant of such an elementary matrix is -1.

3. $\det(AD) = a$ since the matrix C corresponds to an elementary where two pairs of rows are swapped and the determinant of such an elementary matrix is $-1^2 = 1$.

4. $\det(AE) = 2^4 a$ since E is diagonal and there is a value of 2 on each of the four entries on the diagonal.

5. $\det(AF) = 0 \cdot a = 0$ since the determinant of F is zero. There are many ways to see this, but adding -row 1 to row 2 yields all zeros in row 2.