Math 4133 - Linear Algebra

Quiz #10 - 2014.04.09

Solutions

Consider the linear map $T : \mathbb{R}^2 \to \mathbb{R}^4$ defined by $T(\langle x, y\rangle) = \langle x, y, -x, -y\rangle$.

1. Find the matrix A corresponding to the map T.

The matrix A will be a 4×2 matrix, with

$$\begin{bmatrix} a & b \\ c & d \\ e & f \\ g & h \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \\ -x \\ -y \end{bmatrix}$$

Setting up the equations gives

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}$$

2. If $\mathbb{S} = \{\, a\langle 1, 1\rangle \,|\, a \in \mathbb{R}\}$, find a basis for the image subspace $T(\mathbb{S})$.

So all we need to do is compute the image of the basis vector $\langle 1, 1\rangle$, which is $T(\langle 1, 1\rangle) = \langle 1, 1, -1, -1\rangle$. So

$$T(\mathbb{S}) = \{\, a\langle 1, 1, -1, -1\rangle \,|\, a \in \mathbb{R}\}$$

3. Find the inverse image $T^{-1}(\mathbb{K})$ of $\mathbb{K} = \{\, a\langle 1, 1, 0, 0\rangle \,|\, a \in \mathbb{R}\}$.

$T^{-1}(\mathbb{K}) = \left\{ \overrightarrow{0}_2 \right\}$, since no other vector from $\mathbb{R}^2$ will map to a vector of the form $\langle a, a, \rangle$ under the map T.