

Math 4133 - Linear Algebra

Quiz #11 - 2014.04.23

Solutions

Consider the matrix $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ -2 & 3 \\ 0 & 1 \end{bmatrix}$.

1. Determine the linear map T corresponding to the matrix A .

$$T(\langle x, y \rangle) = \langle x, x + y, -2x + 3y, y \rangle$$

2. State the domain and codomain of T .

$$T : \mathbb{R}^2 \longrightarrow \mathbb{R}^4$$

3. Compute $\text{Im}(T)$. Express your answer as the span of a set of vectors in the codomain.

Using problem 1, we have that

$$T(\langle x, y \rangle) = x\langle 1, 1, -2, 0 \rangle + y\langle 0, 1, 3, 1 \rangle$$

So

$$\text{Im}(T) = \text{span} \{ \langle 1, 1, -2, 0 \rangle, \langle 0, 1, 3, 1 \rangle \}$$

4. Compute $\text{Ker}(T)$. Express your answer as the span of a set of vectors in the domain.

Here we need the pre-image of $\vec{0}_4$, however looking at the definition of T as given in problem 1, we have that $x = y = 0$, so

$$\text{Ker}(T) = \{ \vec{0}_2 \}$$

5. Compute $\dim(\text{Im}(T)) + \dim(\text{Ker}(T))$.

$$\dim(\text{Im}(T)) + \dim(\text{Ker}(T)) = 2 + 0 = 2.$$