CS 3513 - Numerical Analysis

Homework #1 - 2006.08.23

Due Date - 2006.08.30

Name: ___________________________

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1. Use the Intermediate Value Theorem to prove that $f(c) = 0$ for $c \in (0, 1)$.

a) $f(x) = x^3 - 4x + 1$

b) $f(x) = 5\cos(\pi x) - 4$

c) $f(x) = 8x^4 - 8x^2 + 1$

2. Find c satisfying the Mean Value Theorem for $f(x)$ on the interval $[0, 1]$.

a) $f(x) = e^x$

b) $f(x) = x^2$

c) $f(x) = \frac{1}{x+1}$

3. Find c satisfying the Mean Value Theorem for Integrals with $f(x)$, $g(x)$ in the interval $[0, 1]$.

a) $f(x) = x$, $g(x) = x$

b) $f(x) = x^2$, $g(x) = x$

c) $f(x) = x$, $g(x) = e^x$

4. Find the Taylor polynomial of degree 2 about the point $x = 0$ for the following functions:

a) $f(x) = e^{x^2}$

b) $f(x) = \cos(5x)$

c) $f(x) = \frac{1}{x+1}$

5. Find the Taylor polynomial of degree 5 about the point $x = 0$ for the following functions:

a) $f(x) = e^{x^2}$

b) $f(x) = \cos(2x)$

c) $f(x) = \ln(1 + x)$

d) $f(x) = \sin^2(x)$

6. a) Find the Taylor polynomial of degree 4 for $f(x) = x^{-2}$ about the point $x = 1$.

b) Use the result of a) to approximate $f(0.9)$ and $f(1.1)$.

c) Use the Taylor remainder to find an error formula for the Taylor polynomial. Give error bounds for each of the two approximations made in part b). Which of the two approximations in part b) do you expect to be closer to the correct value?

d) Use a calculator to compare the actual error in each case with your error bound from part c).

7. a) Find the degree 5 Taylor polynomial $P_5(x)$ centered at $x = 0$ for $f(x) = \cos(x)$.

b) Find an upper bound for the error in approximating $f(x) = \cos(x)$ for $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ by $P_5(x)$.

8. A common approximation for $\sqrt{1+x}$ is $1+\frac{1}{2}x$, when x is small. Use the degree 1 Taylor polynomial of $f(x) = \sqrt{1+x}$ with remainder to determine a formula of form $\sqrt{1+x} = 1 + \frac{1}{2}x \pm E$. Evaluate E for the case of approximating $\sqrt{1.02}$. Use a calculator to compare the actual error to your error bound E.