

CS 3513 - Numerical Analysis

Homework #11 - 2006.11.27

Due Date - 2006.12.04

Name: _____

1. Consider the initial value problem

$$\begin{cases} y' = 6y - 6y^2 \\ y(0) = \frac{1}{2} \\ y \in [0, 20]. \end{cases}$$

a) If $y(0) = \frac{1}{2}$, what values must $y(t)$ lie between for all $t \geq 0$?

b) Solve the IVP by separation of variables. Hint: remember partial fractions!

c) Find the formula for w_{i+1} in the Backward Euler Method explicitly, and be sure to use part a) to help finalize your answer.

d) Plot the actual solution $Y(t)$, the Backward Euler Method solution $w(t)$, and the Euler Method solution $z(t)$ all on the same graph.

e) Plot $|Y(t) - z(t)|$ and $|Y(t) - w(t)|$ on the same graph.

2. Consider the initial value problem

$$\begin{cases} x' = \frac{1}{2} \frac{1-t}{t} x \\ x(1) = 1 \\ x \in [1, 15]. \end{cases}$$

a) Solve the IVP by separation of variables.

b) Find the formula for w_{i+1} in the Backward Euler Method explicitly, and be sure to use part a) to help finalize your answer.

c) Plot the actual solution $Y(t)$, the Backward Euler Method solution $w(t)$, and the Euler Method solution $z(t)$ all on the same graph.

d) Plot $|Y(t) - z(t)|$ and $|Y(t) - w(t)|$ on the same graph.