

CS 3513 - Numerical Analysis

Homework #3 - 2006.09.13

Due Date - 2006.09.25

Name: _____

1. Consider the polynomial $P(x) = (x - 1)^4(2x + 1)^2(4x + 7)x$ which has roots at $x = 1$, $x = -\frac{1}{2}$, $x = -\frac{7}{4}$ and $x = 0$. For each root, which method will converge faster, the Bisection Method or Newton's Method?

2. Prove Newton's Method applied to $f(x) = ax + b$ converges in one step.

3. Show that Newton's Method applied to the equation $f(x) = x^2 - A$ produces the iteration used to find $\sqrt{2}$ in the previous homework assignment when $A = 2$.

4. Use Newton's Method to produce a quadratically convergent method for calculating the n^{th} root of a positive number A where n is a positive integer. Prove quadratic convergence.

Computer Problems:

Each of the following equations has one root. Use a) Bisection Method, b) Newton's Method and c) Secant Method to approximate the root to eight correct decimal places. In each case, state your initial condition(s), the number of iterations required to acquire accuracy to eight decimal places, along with the output at each step.

1. $f(x) = x^5 + x - 1$

2. $g(x) = \sin(x) - 6x - 5$

3. $h(x) = e^x + \sin(x) - 4$