CS 3513 - Numerical Analysis

Homework #4 - 2006.09.27

Due Date - 2006.10.06

Solutions

For each of the following matrices, compute (by hand) the inverse of the matrix and its condition number.

1. $A = \begin{bmatrix} 4 & -1 \\ -2 & 6 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} \frac{3}{11} & \frac{1}{22} \\ \frac{1}{11} & \frac{2}{11} \end{bmatrix}$$

Notice that $\| A \|_\infty = 8$ and $\| A^{-1} \|_\infty = \frac{7}{22}$, therefore $cond(A) = \frac{28}{11} \approx 2.55$.

2. $B = \begin{bmatrix} 4 & -1 & 2 \\ 6 & 5 & 2 \\ 0 & -4 & 5 \end{bmatrix}$

$$B^{-1} = \begin{bmatrix} \frac{11}{38} & -\frac{1}{38} & -\frac{2}{19} \\ -\frac{5}{19} & \frac{10}{57} & \frac{2}{57} \\ -\frac{4}{19} & \frac{8}{57} & \frac{13}{57} \end{bmatrix}$$

Notice that $\| B \|_\infty = 13$ and $\| B^{-1} \|_\infty = \frac{11}{19}$, therefore $cond(B) = \frac{143}{19} \approx 7.53$.

3. $C = \begin{bmatrix} \frac{1}{2} & 0 & 40 \\ 7 & 23 & 1 \\ 0 & 19 & -1 \end{bmatrix}$

$$C^{-1} = \begin{bmatrix} -\frac{6}{757} & \frac{760}{5299} & -\frac{920}{5299} \\ \frac{1}{757} & -\frac{1}{10598} & \frac{559}{10598} \\ \frac{19}{757} & -\frac{19}{10598} & \frac{23}{10598} \end{bmatrix}$$

Notice that $\| C \|_\infty = \frac{81}{2}$ and $\| C^{-1} \|_\infty = \frac{246}{757}$, therefore $cond(C) = \frac{9963}{757} \approx 13.16$.

4. $D = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & -9 & 8 & 5 \\ 4 & 3 & 7 & 5 \\ 3 & 2 & 1 & 5 \end{bmatrix}$

$$D^{-1} = \begin{bmatrix} \frac{365}{34} & \frac{59}{34} & -\frac{150}{17} & \frac{84}{17} \\ -\frac{125}{34} & -\frac{23}{34} & \frac{53}{17} & -\frac{29}{17} \\ -\frac{20}{17} & -\frac{3}{17} & \frac{19}{17} & -\frac{12}{17} \\ -\frac{161}{34} & -\frac{25}{34} & \frac{65}{17} & -\frac{33}{17} \end{bmatrix}$$

Notice that $\| D \|_\infty = 22$ and $\| D^{-1} \|_\infty = \frac{446}{17}$, therefore $cond(D) = \frac{9812}{17} = 577.18$.