CS 3513 - Numerical Analysis

Homework #5 - 2006.10.09

Due Date - 2006.10.18

Name: ________________________

Consider the trigonometric function $f(x) = \cos(x)$ on the domain $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

Consider picking (at most) 9 points on the interval and approximate $f(x)$ by the polynomial $P_8(x)$ found by either Lagrange interpolation or Newton's divided differences.

Define the error total error TE of the approximation to be

$$TE = \int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} |f(x) - P_8(x)|\, dx.$$

Your goal for this homework assignment is to find 9 points which makes TE as small as possible on the interval $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$. For each attempt (and I want to see at least 5, plus the Chebyshev polynomial), I want to see the following written down afterwards:

- The 9 points in a list in (x_k, y_k) format.
- The polynomial $P_8(x)$.
- The total error TE.

Note 1: As an example to compute the total error of the function $g(x) = \sin(x)$ and $P(x) = x - \frac{1}{6}x^3$ on the interval $[0, \pi]$, you could use the Matlab command:
$>> [TE, n] = \text{quad}('(\sin(x) - (x - x.\hat{}3/6))', 0, pi, 1.e-10)$
Here the output would be (remember to '*format long*' before running the computations):

$$TE =$$
$$1.12390992587196$$
$$n =$$
$$225$$

where TE is the total error and n is the amount of function evaluations used to arrive at TE. The final entry in the *quad* command is the maximum possible error in the calculation. In the above instance, the TE will be accurate to 10 decimal places.

Note 2: Be consistent with the accuracy of your approximations to $\sin(x_k)$ for $1 \leq k \leq 9$. You should be using the same amount of accuracy for each attempt. Also, be sure that the accuracy in the *quad* command does NOT exceed that accuracy of your approximations to $\sin(x_k)$.