CS 3513 - Numerical Analysis

Homework #9 - 2006.11.08

Due Date - 2006.11.17

Solutions

Show that the following finite-difference formulas are all $\mathcal{O}(h^4)$.

1. $12hf'(x) \approx f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h)$

2. $12hf'(x) \approx -3f(x-h) - 10f(x) + 18f(x+h) - 6f(x+2h) + f(x+3h)$

3. $12hf'(x) \approx -25f(x) + 48f(x+h) - 36f(x+2h) + 16f(x+3h) - 3f(x+4h)$

4. $12hf'(x) \approx 3f(x+h) + 10f(x) - 18f(x-h) + 6f(x-2h) - f(x-3h)$

5. $12hf'(x) \approx 25f(x) - 48f(x-h) + 36f(x-2h) - 16f(x-3h) + 3f(x-4h)$

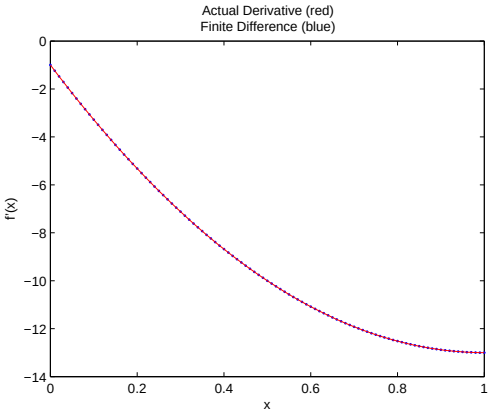
To answer all of the above problems, consider the following expansions:

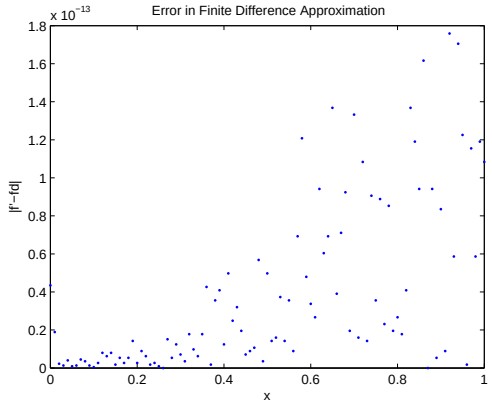
$$
\begin{aligned}
f(x-3h) &= f(x) - 3f'(x)h + f''(x)\frac{9h^2}{2} - f'''(x)\frac{27h^3}{6} + f^{iv}(x)\frac{81h^4}{24} + \dots \\
f(x-2h) &= f(x) - 2f'(x)h + f''(x)\frac{4h^2}{2} - f'''(x)\frac{8h^3}{6} + f^{iv}(x)\frac{16h^4}{24} + \dots \\
f(x-h) &= f(x) - f'(x)h + f''(x)\frac{h^2}{2} - f'''(x)\frac{h^3}{6} + f^{iv}(x)\frac{h^4}{24} + \dots \\
f(x) &= f(x) \\
f(x+h) &= f(x) + f'(x)h + f''(x)\frac{h^2}{2} + f'''(x)\frac{h^3}{6} + f^{iv}(x)\frac{h^4}{24} + \dots \\
f(x+2h) &= f(x) + 2f'(x)h + f''(x)\frac{4h^2}{2} + f'''(x)\frac{8h^3}{6} + f^{iv}(x)\frac{16h^4}{24} + \dots \\
f(x+3h) &= f(x) + 3f'(x)h + f''(x)\frac{9h^2}{2} + f'''(x)\frac{27h^3}{6} + f^{iv}(x)\frac{81h^4}{24} + \dots
\end{aligned}
$$

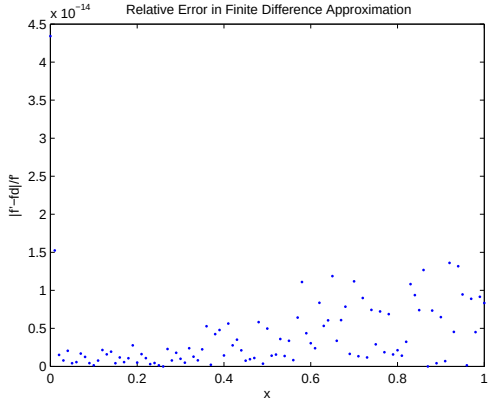
By using the above formulas with the prescribed values from the problems, it is easy to show that the formulas are indeed order

Use the above finite-difference formulas to construct a first derivative program in Matlab. Given a function on an interval $[a, b]$, your program should compute the derivative using the formula from problem 1 on all points except those near the endpoints. Near the endpoints you will have to use the formulas from the other problems. Compare the derivative computed by finite-difference with the actual derivative on the specified interval and plot the difference.

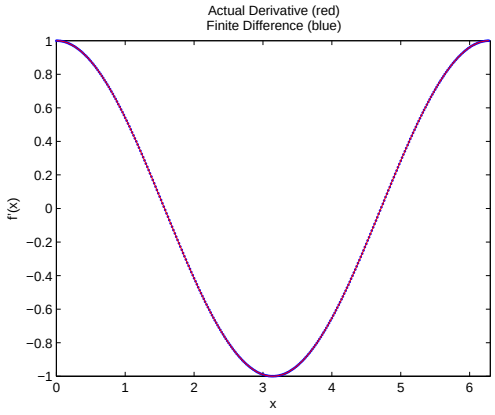
6. $f(x) = 4x^3 - 12x^2 - x + 1$ on $[0, 1]$ with $h = 0.01$

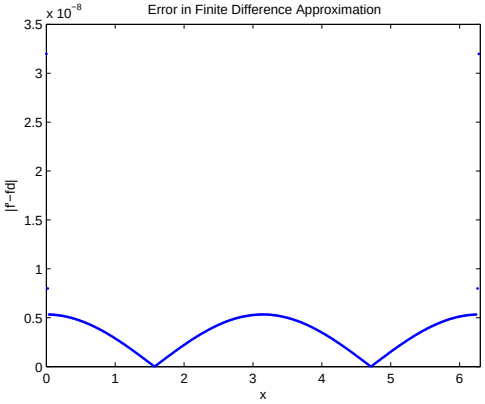


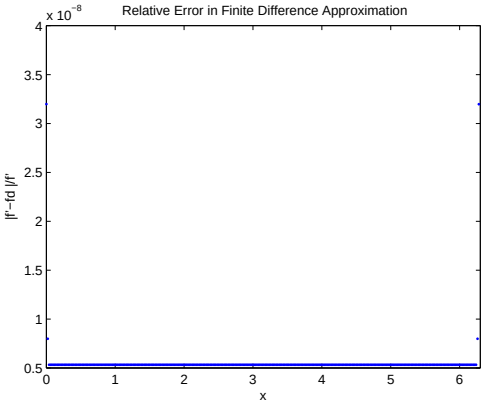




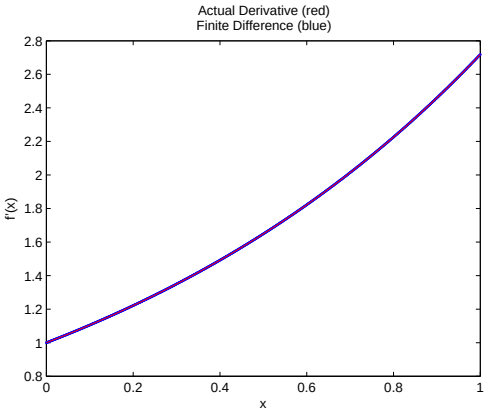
7. $f(x) = \sin(x)$ on $[0, 2\pi]$ with $h = 0.02$

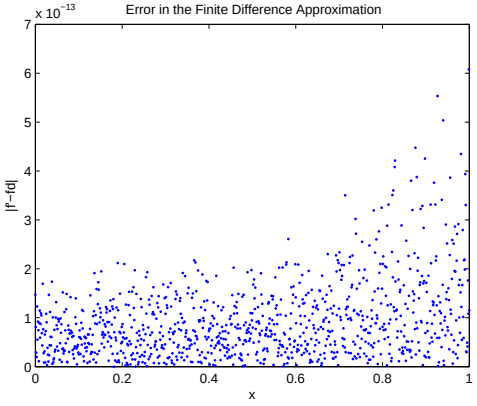


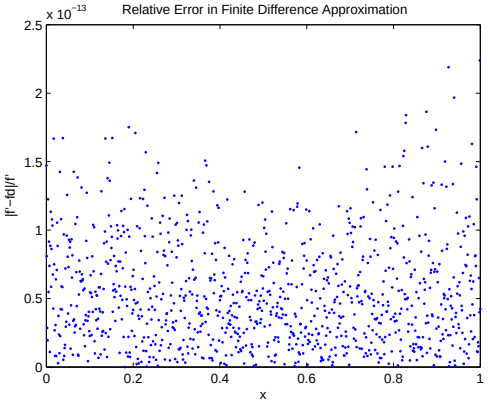




8. $f(x) = e^x$ on $[0, 1]$ with $h = 0.001$







9. $f(x) = xe^x$ on $[0, 2]$ with $h = 0.01$