Project #2
$PA = LU$ Factorization

After studying the mechanics of the $PA = LU$ factorization, it is now time to implement it. You must write a Matlab program which solves a system of n equations and n unknowns using the $PA = LU$ factorization method. This will include:

1. Pivoting where required.

2. Calculating P, L, and U.

3. Storing L on the lower diagonal of U.

4. Back-substitution for solving the equation $L\vec{c} = P\vec{b}$ and then $U\vec{x} = \vec{c}$.

Your will only be allowed to enter the matrix A and the vector b as input for your program. Thus, in your program, you will have to determine the size of the matrix A or vector $\vec{b}$ and thus the number of equations/unknowns in the system for the loops required to perform the $PA = LU$ factorization.

The program should obviously output the solution vector $\vec{x}$, and the permutation matrix P. The matrices L and U can be printed together, as they are stored in one matrix with L being stored below the diagonal.

Run your program on the following pairs of A's and $\vec{b}$'s. Also perform the same operations by hand to ensure that your program is working appropriately. Also, for each $n \times n$ matrix A, compute $cond(A)$. Hand in both your hand calculations and the program results for each pair.

1. $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$

2. $A = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 2 & -2 \\ 2 & 5 & 4 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 1 \\ 7 \\ -1 \end{bmatrix}$

3. $A = \begin{bmatrix} 5 & 4 & 7 \\ 2 & -4 & 3 \\ 2 & 8 & 6 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 2 \\ -5 \\ 4 \end{bmatrix}$

4. $A = \begin{bmatrix} 5 & 4 & 7 \\ 2 & -4 & 3 \\ 2 & 8 & 6 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 5 \\ -4 \\ 2 \end{bmatrix}$

5. $A = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 1 & 3 & 4 & 4 \\ 0 & 2 & 1 & -1 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$