Project #3
Least Squares and Normal Equations

The Idea: Given a set of data points, and the degree of the polynomial that is desired to best fit those points, your program will find the polynomial of best fit.

The Details: Your program should accept three items. First and second are the column vector which contain the x_j and y_j coordinates of the data points (x_j, y_j). The third item is the degree n of the polynomial that you want to best fit the data to.

After you have the data in hand, your program must solve the Least Squares problem using the normal equations. In solving the normal equations, use your $PA = LU$ program from Project #2. Once the system is solved, you are to plot both the data points and the curve. Your program should also output the coefficients in front of each term in your polynomial and the squared error in your polynomial fit to the data.

Remarks:

- Check that the system has more equations than unknowns before beginning.
- Try to call your PA=LU function from inside your Least Squares code and have it return the appropriate solution.
- Remember *the hold* on command when plotting more than one item.

See next page for some examples to use to test your work.s

Examples to test your code on:

1.
$$x = \begin{bmatrix} -3 \\ -1 \\ 0 \\ 1 \\ 3 \end{bmatrix}, \quad y = \begin{bmatrix} 3 \\ 2 \\ 1 \\ -1 \\ -4 \end{bmatrix}, \quad \text{with } n = 1, 2, 3 \text{ and } 4$$

2.
$$x = \begin{bmatrix} 4.0 \\ 4.2 \\ 4.5 \\ 4.7 \\ 5.1 \\ 5.5 \\ 5.9 \\ 6.3 \\ 6.8 \\ 7.1 \end{bmatrix}, \quad y = \begin{bmatrix} 102.56 \\ 113.18 \\ 130.11 \\ 142.05 \\ 167.53 \\ 195.14 \\ 224.87 \\ 256.73 \\ 299.5 \\ 326.72 \end{bmatrix}, \quad \text{with } n = 1, 2, 3, 4 \text{ and } 5$$

3.
$$x = \begin{bmatrix} 0.040 \\ 0.041 \\ 0.055 \\ 0.056 \\ 0.062 \\ 0.071 \\ 0.071 \\ 0.078 \\ 0.082 \\ 0.090 \\ 0.092 \\ 0.100 \\ 0.105 \\ 0.120 \\ 0.123 \\ 0.130 \\ 0.140 \end{bmatrix}, \quad y = \begin{bmatrix} 26.5 \\ 28.1 \\ 25.2 \\ 26.0 \\ 24.0 \\ 25.0 \\ 26.4 \\ 27.2 \\ 25.6 \\ 25.0 \\ 26.8 \\ 24.8 \\ 27.0 \\ 25.0 \\ 27.3 \\ 26.9 \\ 26.2 \end{bmatrix}, \quad \text{with } n = 1, 2, 3 \text{ and } 7$$