Project #4
Runge-Kutta and Systems of ODEs

The Idea: Given a system of ordinary differential equations with initial conditions, your program will solve the system and graph the system in a scene of the user's choosing.

The Details: Your program should accept the following items in one form or another:

1) The equations
2) The initial conditions
3) Initial start time $t = a$
4) End stop time $t = b$
5) The scene (which solution to plot versus which)

After you have the data in hand, your program must solve the system of ordinary differential equations using the Runge-Kutta Method of order 4. After the solution has been computed all the way to time $t = b$, the solution will be plotted in the scene selected by the user.

Remember that if you have the IVP $y' = f(t, y)$, $y(a) = y_a$, then the scene is automatically just t vs. $y(t)$. However if you have the system $y_1' = f_1(t, y_1, y_2)$, $y_2' = f_2(t, y_1, y_2)$, $y_1(a) = y_{1,a}$, $y_2(a) = y_{2,a}$, then you might wish to plot $y_1(t)$ vs. $y_2(t)$ for example.

Remarks: I suggest getting the simpler, one equation program working first. After completing that, modify the code for systems, and then finally do the plotting options.

Definitely take a look at all the sample codes in the book, they will help immensely!

Examples to test your code on:

1.

$$\begin{cases} y_1' = -\frac{1}{2}y_1 + y_2 \\ y_2' = -y_1 - \frac{1}{2}y_2 \\ y_1(0) = 1 \\ y_2(0) = 0 \\ t \in [0, 15] \end{cases}$$

with scenes t vs $y_1(t)$ and $y_1(t)$ vs. $y_2(t)$.

2.

$$\begin{cases} y_1' = -\frac{5}{2}y_1 + y_2 + y_3 \\ y_2' = y_1 - \frac{5}{2}y_2 + y_3 \\ y_3' = y_1 + y_2 - \frac{5}{2}y_3 \\ y_1(0) = 2 \\ y_2(0) = 3 \\ y_3(0) = -1 \\ t \in [0, 15] \end{cases}$$

with scenes t vs $y_3(t)$ and $y_1(t)$ vs. $y_2(t)$ and $y_1(t)$ vs. $y_3(t)$.

3.

$$\begin{cases} y_1' = -2y_1 - y_2 + e^{-t} \\ y_2' = -3y_1 - 2y_2 - e^{-t} \\ y_1(0) = 1 \\ y_2(0) = 1 \\ t \in [0, 25] \end{cases}$$

with scenes t vs $y_1(t)$ and $y_1(t)$ vs. $y_2(t)$.